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AN OUTLINE OF *DE RE* BELIEFS ABOUT NATURAL NUMBERS

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This paper outlines an account of *de re* beliefs about natural numbers. The main claim is that we can have *de re* beliefs about numbers through a notation that fulfills two criteria: surveyability and familiarity. The surveyability condition makes sure that we don't have to deal with a cumbersome notation. Cumbersome notations such as the unary one make it very quickly practically impossible to count or to do simple calculations. The familiarity condition makes sure that we know in a buck-stopping way with which number(s) we are dealing with. Unfamiliar notations such as the base 133 one, don't fulfil this condition. The paper introduces Wittgenstein's points about surveyability and Kripke's about familiarity. It points out that a recent criticism of Kripke by Shapiro misses the point about *de re* beliefs. Following Burge, the paper makes clear that reflection upon *de re* beliefs centers on our basic epistemic abilities to connect our thought to a subject-matter. Thus Shapiro's criticism and own account fail to do justice to this issue.

THE CONCEPT OF PROOF

Matthias Baaz

Vienna, Austria

The concept of proof is the most fundamental notion in mathematics. The Hilbertian revolution at the beginning of the 20th century is based on an atomic notion of proof that is the foundation of the axiomatic method: a proof is a finite sequence of formulas A_1, \dots, A_n such that each A_i is an instance of an axiom or follows by direct application of a rule from A_1, \dots, A_k with $k < i$. While no scientific revolution is total, there is a tendency to disregard all alternatives to this successful method. In this lecture we discuss more global notions of proof where subproofs are not necessarily proofs themselves. Examples are among others

1. proto-proofs in the sense of Euler's famous solution to the Basel problem, which uses analogical reasoning and where additional external justifications are necessary.
2. circular notions of proof, where the concept of proof itself incorporates induction
3. sound proofs based on locally unsound rules
4. proofs based on abstract proof descriptions prominent e.g. in Bourbaki, where only the choice of a suitable result makes a verification possible.

We discuss the benefits of these alternative concepts and the possibility that innovative concepts of proof tailored to the problems in question might lead to strong mathematical results and constitute a novel area of Proof Theory.

A COLLECTIVIST INTERPRETATION OF THE LOGICAL MUST: THE LATER WITTGENSTEIN'S VIEW ON LOGIC

Zhien Bei & Shier Ju

Guangzhou, China

The primary purpose of this essay is to put forward a collectivist interpretation of the logical must from the later Wittgenstein's new treatment of logic and the collectivist account of rule-following. Firstly, the later Wittgenstein's treatment of logic is described through three steps, namely, essence removed, rule-governed activity emphasized and form of life introduced; and then its relation to rule-following is shown. Secondly,

Wittgenstein's interpretation of the "hardness of logical must" is analyzed. According to this interpretation, the social-cultural factor and function-oriented character of ordinary logical activity are taken into account. Based on these considerations, the logical must should be regarded as the collectivist normativity of rules. Furthermore, by arguing against the individualist challenge we defend our collectivist interpretation. In addition, we explain how social consensus and group interaction play a central role in the establishment of the normativity of logical rules. Consequently, we conclude that such a collectivist interpretation of the logical must is conducive to enlarging the scope and enriching the methods in logic study.

WITTGENSTEIN, DIE GRUNDLAGEN DER MATHEMATIK UND HEGEL

Alexander Berg

Prag, Tschechische Republik

Einem von Wittgenstein selbst vorgeschlagenen Lexikon-Eintrag zufolge hat dieser seine Beschäftigung mit den Grundlagen der Mathematik als dasjenige betrachtet, was die philosophische Nachwelt über sein Denken zu wissen braucht. In den *Vorlesungen über die Grundlagen der Mathematik* von 1939 untersucht Wittgenstein wiederum, was genau er als das Wesentliche dieser Beschäftigung betrachtet. Dabei stellt sich heraus, dass es Wittgenstein um eine Betonung der Unterschiede bzw. Differenzen beim Umgang mit (mathematischen) Begriffen geht, welche er besonders einem Einheits- oder Identitätsdenken der Begriffe gegenüberstellt.

Die Charakterisierung dieses Gegensatzes erinnert in ihrer Struktur stark an eine weitere Überlegung Wittgensteins, in welcher er etwas später ebenfalls sein Denken über seine Differenzierungsbemühungen charakterisiert – nur erfolgt in diesem Fall die Abgrenzung seiner *Differenzphilosophie* gegen eine *Identitätsphilosophie* G. W. F. Hegels.

Die Untersuchung versucht zu klären, auf welche Weise Wittgenstein ursprünglich zu diesen Überlegungen gekommen ist und darauf aufbauend, was sie vor dem Hintergrund seines eigenen Philosophie-Verständnisses bedeuten.

ON KRIPKENSTEINIANS: RULES, GRAMMAR & CHESS

Eduardo Bermúdez Barrera, René J. Campis C. & Osvaldo Orozco Méndez

Barranquilla, Colombia

This contribution further develops Bermúdez ("Wittgenstein, Language and Chess", *Papers of the 29th Wittgenstein Symposium*, E. Leinfellner et al. (eds.), 2006, 29–32). We claim that in order to achieve a thorough comprehension of the philosophy of language and the theory of meaning of Wittgenstein, basic knowledge of chess is required, an issue which was completely missed by philosophers like Moore, Kripke, Kripkensteinians and others. Wittgenstein's philosophy developed altogether with his comprehension of the rules and the metaphor of chess. Examples are presented.

ADDING 4.0241 TO TLP

Franz Berto

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TLP 4.024 inspired the dominant semantics of our time: truth-conditional semantics. This focusses on possible worlds: the meaning of p is the set of worlds where p is true.

As S. Yablo points out, however, there are defects in this conception and what is needed is an “independent factor in meaning, constrained but not determined by truth-conditions” (*Aboutness*, Princeton University Press, 2014, 2). In other words, we need a way of understanding how sentences can be true in different ways in one and the same world. Such different ways of being true are to be understood in terms of content mereologies, world-partitions, aboutness, truthmaking.

In this talk I suggest a “missing” TLP comment which, had it been included in the TLP, would have helped semantics get this right from the start. This is my “4.0241”: “Knowing what is the case if a proposition is true is knowing its ways of being true”. This is to say that knowing what is the case when a proposition is true is a matter of knowing a proposition’s truth possibilities *and*, what we now call, its topic, or subject matter.

I show that the famous “fundamental thought” that “the ‘logical constants’ do not represent” (4.0312), can be understood in terms of ways-based views of meaning. These also help with puzzling claims like 5.122, “If p follows from q , the sense of [p] is contained in the sense of [q]”, which hint at a conception of entailment that combines truth-preservation with the preservation of topicality, or of ways of being true.

WITTGENSTEIN’S THREEFOLD VIEW ON PROOFS

Kim-Erik Berts

Abo, Finland

In his remarks on the philosophy of mathematics, Wittgenstein emphasizes three aspects of proofs: (1) that proofs contribute to the meaning of the concepts involved in theorems, (2) that there is a fundamental difference between proofs and experiments, and (3) that proofs must be surveyable. He sees these aspects as inseparable from each other and as features of the grammar of proofs. This paper illuminates Wittgenstein’s view on mathematical proofs by discussing these three aspects of proofs and by considering how a specific proof – of the Bolzano-Weierstrass theorem – exhibits them.

CLASSICAL PROPOSITIONAL LOGIC IN THE TRACTATUS VERSUS ELSEWHERE

Jean-Yves Beziau

Rio de Janeiro, Brazil

In this talk I consider the basic technical features of classical propositional logic (CPL) as presented in the *Tractatus*.

I compare Wittgenstein’s presentation with: (1) CPL as presented by A.N. Whitehead and Bertrand Russell in the first edition of their *Principia Mathematica* (Cambridge University Press, 1910); (2) Emil Post’s paper published the same year as the *Tractatus*, “Introduction to a General Theory of Propositions” (*American Journal of Mathematics*, 43, 1921, pp.163–185); and (3) the mathematical vision we have, nowadays, of CPL.

I emphasize the fact that there is a definition of semantical consequence (5.11) in the *Tractatus*, and that if we take paragraph 5.141 literally, then propositional logic is considered as a Boolean algebra (without proof: the proof being given only later, by Tarski).

This work is a contribution to the history and development of modern logic, a follow-up to my previous paper, “History of Truth-Values”, (in D.M. Gabbay, F.J. Pelletier and J. Woods (eds.) *Handbook of the History of Logic, Vol. 11 – Logic: A History of its Central Concepts*, Elsevier, Amsterdam, 2012, 233–305).

„WARUM GESCHIEHT DIES? (...) WEIL ES FURCHTBAR IST“ – WITTGENSTEIN’S KONZEPT DER BESCHREIBUNG IN SEINEN BEMERKUNGEN ÜBER FRAZERS „THE GOLDEN BOUGH“

Anna-Maria Brandtner

München, Deutschland

In dieser Untersuchung werde ich die zwei wichtigsten Kritikpunkte Wittgensteins an Frazers Erklärungsversuchen von Bräuchen in dessen *Bemerkungen über Frazers „The Golden Bough“* erläutern. Der Schwerpunkt der Arbeit liegt in der Fragestellung, wie Wittgenstein die zwei daraus resultierenden Forderungen selbst mit seinem Beschreibungsmodell inkorporieren möchte. Einerseits sollten dabei Beschreibungen die Kontingenz von Erklärungen vermeiden, indem sie sich ausschließlich auf Wissen stützten und darum in Gänze auf Hypothesen verzichteten. Andererseits dürfe der emotional-psychologische Nutzen von Bräuchen nicht vernachlässigt werden, welcher jene überhaupt erst auszeichne. Dieser sei aber nicht anhand von Brauchhandlungen, sondern nur durch das Verständnis des inneren Geistes eines Brauches festzustellen. Die Erfüllung beider Forderungen birgt eine erkenntnistheoretische Schwierigkeit: Wie ist es möglich, genuines Wissen über das Innere eines Brauches zu erlangen, wenn dieses nicht durch äußere Merkmale generiert werden könne? Zwar beantwortet Wittgenstein jene Frage, doch werde ich in einem letzten Schritt aufzeigen, dass seine Antwort selbst wiederum höchst problematisch ist.

WHITEHEAD AND WITTGENSTEIN ON THE THEORY OF TYPES, SYMBOLISM, AND MATHEMATICS

Romain Büchi

Zurich, Switzerland

This paper is an attempt to trace Whitehead’s influence on the young Wittgenstein. After a brief sketch of their professional and personal relationship, three tracks are followed. First, Wittgenstein’s criticism of the axiom of reducibility, expressed in a letter to Russell, is related to the “Prefatory Statement of Symbolic Conventions” that appeared at the beginning of the second volume of *Principia Mathematica* and in which Whitehead offers substantial revisions to the theory of types expounded in the first volume. Whitehead’s idea of introducing symbolic forms of propositional functions as the real bearers of typical ambiguity parallels, as argued in the second section, Wittgenstein’s later distinction between sign and symbol. It is also shown that the other tenet of Wittgenstein’s symbolic turn – i.e. the conception of symbols, and signs, as facts rather than complexes – can already be found in Whitehead’s *Treatise on Universal Algebra*. The last section, further explores the remarkable fact that the picture of mathematics offered in the *Tractatus* is in many respects much more akin to what can be found in the *Treatise on Universal Algebra* than in *Principia Mathematica*.

NATURAL NUMBERS AS TROPES

Claudio Costa

Natal, Rio Grande do Norte, Brazil

A natural number can only be a trope if there is a way to understand it as some kind of spatiotemporally localizable property. In this paper, a strategy is developed in order to explain applied natural numbers as having this property. It

is also shown how this strategy can be extended to what is common to all equinumerous applied natural numbers, namely, to the abstractly considered natural numbers. By doing this, the author is not trying to define numbers by means of tropes but to identify numbers with tropes.

A NATURALISTIC JUSTIFICATION OF THE GENERIC MULTIVERSE WITH A CORE

Matteo de Ceglie

Salzburg, Austria

In this paper, I argue that a naturalist approach in philosophy of mathematics justifies a pluralist conception of set theory. For the pluralist, there is not a Single Universe, but there is rather a Multiverse, composed by a plurality of universes generated by various set theories. In order to justify a pluralistic approach to sets, I apply the two naturalistic principles developed by Penelope Maddy (cfr. Maddy 1997), *Naturalism in Mathematics*), UNIFY and MAXIMIZE, and analyse through them the potential of the set theoretic multiverse to be the best framework for mathematical practice. According to UNIFY, an adequate set theory should be foundational, in the sense that it should allow one to represent all the currently accepted mathematical theories. As for MAXIMIZE, this states that any adequate set theory should be as powerful as possible, allowing one to prove as many results and isomorphisms as possible. In a recent paper, Maddy (cfr. Maddy 2017), "Set-theoretic Foundations") has argued that this two principle justify ZFC as the best framework for mathematical practice. I argue that, *pace* Maddy, these two principles justify a multiverse conception of set theory, more precisely, the generic multiverse with a core (GM_H).

SURPRISES IN LOGIC: DYNAMIC FORMALITY MEETS INTERACTIVE COMPOSITIONALITY

Elena Dragalina-Chernaya

Moscow, Russia

Logic should explain the contribution of structural complexity to inference. In what sense (if any) logical complexity may be considered as compositional complexity? This paper attempts to provide a taxonomy for the variety of formality in logic. Focusing on Ludwig Wittgenstein's claim that "there can never be surprises in logic" I propose an interactive interpretation of compositionality as dynamic formality. I suggest that the interactive dynamic of information processing provides a unified framework for dealing with binary semantical phenomena. Firstly, I compare model-theoretical and game-theoretical approaches to binary quantifiers. Secondly, I address Wittgenstein's "puzzle proposition" that "there can be a bluish green but not a reddish green". Surprisingly, recent neuropsychological experiments have shown that "unimaginable" binary colours (i.e. reddish green and yellowish blue) can be perceived under artificial laboratory conditions. To answer this experimental challenge, I develop a game-theoretical interpretation of the colour opponency violation in payoff independence logic. Game-theoretical semantics for opponent-processing matches with the experimental data due to considering the fact that the binary colours are affected by the context, i.e. surroundings and conditions of patches of colour perception (e.g. image stabilization and equal luminance). Finally, I argue for the advantages of game-theoretical semantics as an attempt of modelling a balance between compositionality and the context principle.

THE PROCEDURAL CONCEPTION OF FREGEAN SENSE

Günther Eder

Salzburg, Austria

In his seminal paper "On Sense and Reference" (Engl. in: G. Frege, 1984, *Collected Papers on Mathematics, Logic, and Philosophy*, Basil Blackwell, 157–77), Gottlob Frege famously introduced the distinction between sense and reference of an expression. While Frege's notion of reference has been mostly regarded as uncontroversial and eventually found its way into formal semantic theorizing, the notion of sense proved to be more elusive and notoriously hard to pin down in precise terms. In this talk, yet another attempt is made to explicate Frege's notion of sense by building on some of Frege's metaphors where the sense of an expression is described as a "way of arriving" at its referent. Following up on ideas of earlier scholars of how this is to be understood, the sense of an expression will be identified with a procedure to determine its referent. The main goal of this talk is to make this informal idea precise for sufficiently rich (formal) languages. In order to achieve this, a simple semantic model that was introduced by John Horty (*Frege on Definitions: A Case Study of Semantic Content*, Oxford University Press, 2007) will be refined and further generalized.

The resulting formal notion of sense will be compared to other notions of semantic content currently on the market, and possible applications to the semantic paradoxes will be discussed.

ON THE INFINITE: IN-POTENTIA

Susan Edwards-McKie

Cambridge, UK

I shall build on my paper "Following a Rule without the Platonic Equivalent: Wittgenstein's Intentionality and Generality" (in *The Philosophy of Perception and Observation: Contributions to the 40th International Wittgenstein Symposium*, 2017) which explored the relation of the iterative operation to the potential infinite. Firstly, focussing on the principle of contextuality, I look at similarities and differences between Wittgenstein and Frege, which harmonize in interesting ways with the Dedekind cut and the actual infinite when viewed from the Fregean standpoint, but form a distinctly non-Dedekind paradigm when viewed from Wittgenstein's standpoint. I shall consider the principle of composition through Frege's critical question to Wittgenstein: "What cements things together?" with questions of range, part and whole. Wittgenstein's idea that it is the *Eigenschaft* of "5" to be the *Gegenstand* of the rule " $3 + 2 = 5$ " is contrasted with Frege's Platonic work in "Der Gedanke". Questions of the role of the Tractarian *Gegenstand* in developing rules of iteration, compositionality and use, and McGuinness' and Pears' retranslation of *Sachverhalte* from "atomic fact" to that which is in-potentia (state of affairs) is briefly highlighted. Lastly, I provide a *Nachlass* discovery which suggests Wittgenstein continued to work on the highly mathematical TS 222, which later becomes *Remarks on the Foundations of Mathematics*, later than hitherto thought by scholars, precisely in the areas we have considered in the previous sections.

LOOKING FOR WITTGENSTEIN'S LOGICAL ATOMISM

Landon D. C. Elkind

Iowa City, Iowa, USA

Wittgenstein scholars have typically looked for Wittgenstein's logical atomism in the *Tractatus*. I rather argue that the *Tractatus* is the wrong place to look: Wittgenstein's logical atomism comes earlier, closer to his time in Cambridge studying under Russell. This is because by November 1913, Wittgenstein embraces the thesis that all logics theses are tautologies: this constitutes his abandonment of the crucial tenet of logical atomism, namely, that logic is a genuine science. That logic is a genuine science is what empowers the search for logical forms that is characteristic of, and critical to, logical atomism. Wittgenstein's philosophy of logic in the *Tractatus*, particularly his rejection of logic's status as a genuine science, marks his abandonment of logical atomism. Fleshing this out produces an informative chronology of Wittgenstein's pre-*Tractatus* philosophy of logic.

ETHICS AS TRANSCENDENTAL IN THE TRACTATUS LOGICO-PHILOSOPHICUS: RESORTING TO LOGIC

Jordi Fairhurst

Palma de Mallorca, Spain

The aim of this paper is to analyze the transcendental character of ethics as defended by Wittgenstein in the *Tractatus Logico-Philosophicus*. Initially, it considers the Transcendental Reading's understanding of the transcendental character of ethics as advancing a transcendental willing subject condition of ethics and demonstrates the insufficiencies of this proposal. Subsequently, it advances an alternative understanding of the transcendental character of ethics resorting to, and clarifying, the transcendental character of logic, while avoiding some of the misconceptions present in various defenses of this approach. I will argue that whilst logic is the condition for the possibility of picturing the world, ethics is the condition for the possibility to value or evaluate, in an absolute sense, the world.

ONE-STEP, TWO-STEP: DIFFERENT DIAGONAL ARGUMENTS

Zhao Fan

Christchurch, Canterbury, New Zealand

This paper examines Felix Mühlhölzer's claim that Wittgenstein's 1947 remark on diagonal argument presents a one-step version of Cantor's two-step diagonal argument. After clarifying Wittgenstein's remark, I show Mühlhölzer's interpretation is not the same as Wittgenstein's own conclusion. Nevertheless, I explain this interpretation can be viewed as a plausible implication of Wittgenstein's conclusion in terms of the notion of a Turing machine. By doing so, I show the one-step and two-step are different, but not incompatible diagonal arguments. And this will shed new light in interpreting Wittgenstein. Several misreadings are also discussed along the way.

„MAN KANN DIE MENSCHEN NICHT ZUM GUTEN FÜHREN“ – ZUR LOGIK DES MORALISCHEN URTEILS BEI WITTGENSTEIN UND HEGEL

Oliver Feldmann

Wien, Österreich

Die in jüngerer Zeit beliebt gewordenen Versuche, Wittgenstein und Hegel in ein Nahverhältnis zu bringen, sind müßig. Die beiden könnten kaum gegensätzlicher sein, und Wittgenstein hat diesen Gegensatz auch selbst festgehalten mit seinem Pochen auf den Standpunkt der „Verschiedenheit“ gegenüber jenem der Allgemeinheit bei Hegel (*Recollections of Wittgenstein*, hrsg. v. Rush Rhees, Oxford 1981, S. 157).

Dennoch gibt es einen zentralen Punkt, an dem die beiden Denker sich begegnen – und das sogar zweifach. Es ist der Gedanke der „Grenze“, an dem Wittgenstein sich immer wieder entlang bewegt. Und der bei Hegel stets Kritik kennzeichnet.

Wittgensteins Diktum, dass „das Gute“ „außerhalb des Tatsachenraums“ liege, welcher ob der Zufälligkeit seines „So-Seins“ alles sinnvolle Sprechen über „Höheres“ verunmögliche (*Tractatus*, 6.41–6.42), findet seinen Widerhall in dem Urteil Hegels über Kant: „Die vollendete Moralität muss ein Jenseits bleiben; denn die Moralität setzt die Verschiedenheit des besonderen und allgemeinen Willens voraus“ (*Vorlesungen über die Geschichte der Philosophie*, Werkausgabe, Frankfurt/M. 1971, Bd. 20, S. 369).

Während Wittgenstein glaubt, alles eigentlich „Wichtige“ sei genau dort anzusiedeln und unzugänglich, ergeben sich bei Hegel interessante Einsichten über die Verwandtschaft von Moral und Heuchelei und den notwendigen Idealismus der Moralphilosophie.

STRUCTURAL COMPLEXITY: LINGUISTIC APPLICATIONS AND THE SPECIAL STATUS OF DENYING PROPOSITIONS

Fenk August

Klagenfurt, Austria

The paper starts from the general concepts of structural complexity (Simon 1962, „The Architecture of Complexity“) and information entropy (Shannon 1948, „A Mathematical Theory of Communication“) and turns then to some applications and results in quantitative linguistics, such as a „constant“ flow of linguistic information and cross-linguistic correlations indicating complexity trade-offs between aggregation levels in the language system. Next, I focus on denying propositions. They are, also cognitively, more complex than their counterpart and of particular interest in the context of (inductive) logic: A null hypothesis, for example, explicitly denies the existence of the effect or difference claimed in the alternative hypothesis. Semantically more complex propositions may, less obviously, imply such denial: „Only humans have trait x“ implies that „no other animal has this particular trait“ (Hauser *et al.* 2002, „The Faculty of Language: What Is It, Who Has It, and How Did It Evolve?“). Human uniqueness claims concerning symbolic communication and recursive hierarchical processing will be discussed with respect to a fundamental asymmetry between positive and negative evidence.

THE CASE AGAINST BIVALENCE IN LOGIC AND BEYOND

Gregor Flock

Vienna, Austria

In this paper I argue that bivalent systems in logic and beyond are often too inexact and that bivalence about truth-values and other value spectra will consequently and generally need to be replaced with multivalence. I begin arguing for that conclusion by showing how, contrary to an at least potential misconception and framed in set theoretical terms, the somewhat reconceived law of excluded middle does not entail bivalence but is entirely compatible with multivalence too. From these logico-semantic considerations I then move on to some ontological and empirical considerations in favor of multivalence and against bivalence that can be traced back to Łukasiewicz or to the Vienna Circle's logical empiricism. I conclude by pointing out that the paradigm shift from bivalence to multivalence has already occurred in the logic-related and so-called psychology of reasoning and by going through some options of how the bivalence-implying values of "true" and "false" can be adapted to a framework of "quantified multivalence".

WITTGENSTEIN AND TURING

Juliet Floyd

Boston, Massachusetts, USA

A Just-So story, intended as plausible philosophical reconstruction, of the mutual impact of Wittgenstein and Turing upon one another. Recognizably Wittgensteinian features of Turing's diagonal argumentation and machine-model of human computation in "On Computable Numbers, with an Application to the *Entscheidungsproblem*" (*Proceedings of the London Mathematical Society*, 1936/7, 230–265) and his argumentation in "Computing Machinery and Intelligence" (1950) are drawn out, emphasizing the anti-psychologistic, ordinary language and intersubjectivist elements of Turing's conception. These were indebted, on this story, to exposure to Wittgenstein's lectures and dictations. Next Wittgenstein's manuscripts on the foundations of mathematics 1934–1942 are interpreted in light of the impact of Turing's analysis of logic upon them. Themes will include the emergence of rule-following issues, the notion of *Lebensform*, and anti-psychologism. The payoff is a novel and more adequate characterization, both of Turing's philosophy of logic and of Wittgenstein's writing on mathematics.

ON NOT EXPLAINING ANYTHING AWAY

Craig Fox & Eran Guter

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Yezreel Valley, Israel

In this paper we explain Wittgenstein's claim in a 1933 lecture that "aesthetics like psychoanalysis doesn't explain anything away." The discussions of aesthetics are distinctive: Wittgenstein gives a positive account of the relationship between aesthetics and psychoanalysis, as contrasted with psychology. And we follow not only his distinction between cause and reason, but also between hypothesis and representation, along with his use of the notion of ideals as facilitators of aesthetic discourse. We conclude that aesthetics, like psychoanalysis, preserves the very fine phenomena in their fullness.

A KIERKEGAARDIAN INFLUENCE ON THE TRACTATUS?

Mélissa Fox-Muratton

Clermont-Ferrand, France

This article examines the context in which themes of interiority and critique of language emerge within Viennese culture in the early 20th century, in order to sketch out the ways in which the reception of Kierkegaard's philosophy may have impacted Wittgenstein's *Tractatus*. Drawing on the context of Theodor Haecker's philosophy of interiority, Fritz Mauthner's *Sprachkritik*, and the general critique of institutionalisation and cultural decadence at the time, we show that the reception of Kierkegaard's work presents a particular and somewhat erroneous view of Kierkegaard's thought which may have influenced the early Wittgenstein. Reading the relationship between Wittgenstein and Kierkegaard through this context offers a means of getting beyond comparative studies focussing on paradox and ineffability.

ON THE SIZE OF INFINITE SETS: SOME WITTGENSTEINIAN THEMES

Pasquale Frascolla

Potenza, Italy

This talk examines Wittgenstein's remarks on Cantor's diagonal proof of the uncountability of the set of real numbers (*Remarks on the Foundations of Mathematics*, Part II, and *Lectures on the Foundations of Mathematics*, Cambridge, 1939).

The standard account of size for sets (which establishes a tight connection between the comparison of the sizes of any two sets A and B and the existence of certain functions from A to B) is contrasted with the alternative scenario in which Wittgenstein proposes to place Cantor's proof.

First, I consider the similarity of Wittgenstein's stance to the constructivist interpretation of Cantor's Theorem, given by Poincaré and Brouwer.

Second, I discuss Georg Kreisel's ironic comment on Wittgenstein's statement that Cantor's Theorem, through his diagonal proof, gives sense to the expression, the "expansion which is different from all the expansions in a system" (cf. G. Kreisel, "Wittgenstein's *Remarks on the Foundations of Mathematics*", *The British Journal for the Philosophy of Science*, 9, 1958, 135–158).

I trace the rationale of Wittgenstein's position back to his conception of the relationship between mathematics and meaning, and to his peculiar views on semantic normativity.

Lastly, I examine Wittgenstein's use of the criterion of applicability of mathematical theorems outside mathematics as the basis for a substantial rejection of transfinite arithmetic, and tackle the inevitable problem of how to make that rejection consistent with Wittgenstein's quietist meta-philosophical attitude.

LOGISCHE MEHRDEUTIGKEIT UND LOGISCHE UNBESTIMMTHEIT

Georg Friedrich

Graz, Österreich

Die logische Mehrdeutigkeit ist eine von vielen Arten der Mehrdeutigkeit, sie tritt auf, wenn man einen Satz einer natürlichen Sprache in eine formale Sprache überträgt, diesen also formalisiert. Ich werde zunächst versuchen, den Begriff der logischen Mehrdeutigkeit anhand einiger Beispiele verständlich zu machen, um die logische Mehrdeutigkeit dann mithilfe eines Kriteriums von logischer

Unbestimmtheit zu unterscheiden: Ein Satz p einer natürlichen Sprache L_1 sei genau dann *logisch mehrdeutig*, wenn es mindestens zwei Formalisierungen φ_1 und φ_2 von p in der formalen Sprache L_2 gibt, die gleichermaßen als korrekt angesehen werden können und zwischen denen sich ein rationaler Sprecher, der Satz p behauptet, entscheiden muss. Was den Begriff der Mehrdeutigkeit im Allgemeinen betrifft, so gehe ich erstens davon aus, dass dieser nur dann adäquat charakterisiert werden kann, wenn man ihn auf einer pragmatischen Ebene zwischen Sprecher und Hörer einer Äußerung betrachtet, und zweitens, dass nicht bloß das mehrdeutig genannt werden sollte, was mehrere Bedeutungen hat, sondern das, was mehrere Deutungen zulässt.

WITTGENSTEIN AND MARX ON PHILOSOPHY

Dimitris Gakis

Leuven, Belgium

The paper explores some of the affinities that can be discerned between Wittgenstein and Marx from a metaphilosophical point of view. Starting with a discussion of their respective approaches to the potentially transformational character of philosophy, it then engages with the issue of the relation between philosophy and everyday life. Subsequently, it moves to a discussion of Wittgenstein's and Marx's views on the end of philosophy (as both goal and termination) and ends with highlighting later Wittgenstein's therapeutic philosophy as a potential contribution to the project of human emancipation.

WITTGENSTEIN ON DREAMS AND MEANING

Heather J. Gert

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Between §232 and §378 of *Remarks on the Philosophy of Psychology* 1, Wittgenstein makes a number of intriguing remarks that connect dreaming, understanding, and meaning coming into one's mind. In this paper I consider a few passages in which he uses an analogy between *dreaming* and *meaning coming into one's mind* to illustrate his thoughts about understanding one another's utterances.

WITTGENSTEIN ON CANTOR'S PROOF

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Cantor's proof that the reals cannot be enumerated is simple and elegant; indeed, for many mathematicians and logicians, among the most elegant in mathematics. However, even if we accept the proof, and even if as an Archimedean point it supports tomes of mathematical theory, there is a question that lingers on, and demands clarification: What does Cantor's proof *show*? In this paper, I will attempt to clarify what Cantor's proof shows. This is largely an interpretive issue, and one that I think has not been adequately dealt with in the literature. One of few places where it is addressed is Appendix II of Wittgenstein's *Remarks on the Foundations of Mathematics*. Regrettably, however, Wittgenstein's discussion is brief and opaque. Nevertheless, he offers many important insights on the proof, and in what follows, I will use his critique to lay the groundwork for my own discussion.

FREE LOGIC AND QUANTIFIED ARGUMENT CALCULUS

Norbert Gratzl & Edi Pavlovic

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The Quantified Argument Calculus (or Quarc for short) is a novel and peculiar system on quantified logic, particularly in its treatment of non-emptiness of unary predicates, as in Quarc unary predicates are never empty, and singular terms denote. Moreover, and as a consequence of this, the universally quantified formulas entail their corresponding particular ones, similar to existential import. But at the same time, Quarc eschews talk of existence entirely by having a particular quantifier instead of an existential one. To bring it back into consideration, we explicitly introduce the existence predicate, and modify the rules to make the existence assumption obvious. This leads to a version of positive free logic. A question that arises at this point, given that we are interested in free logic, is what happens when we remove the existence assumption on singular terms; here we can quite naturally choose the negative free logic framework. In this paper we shall therefore investigate interrelations between Quarc and free logic (especially with its positive and negative variant), and approach these interrelations with proof-theoretic methods.

THE "UNREASONABLE EFFECTIVENESS OF MATHEMATICS" IN BIOLOGY AND THE FALLACY FROM COMPLEXITY

Gregor Paul Greslehner

Salzburg, Austria

A frequently put forward argument claims that biological systems are too complex for mathematical methods to be fruitfully applied. I argue that this argument from complexity is a fallacy. To the contrary, it is exactly the complexity of biological systems which calls for the use of mathematical methods. While some research strategies in molecular biology used to be less accessible for mathematical analysis, the emergence of systems biology as a scientific discipline is the most recent example of successful and effective applications of mathematical methods in biology. In today's scientific practice, there is evidence for the "unreasonable effectiveness of mathematics" in biology, differing from traditional mathematical biology, thus giving new support to a notion that still cannot be taken for granted.

ONLINE TEXT SEARCH OF THE ENTIRE WITTGENSTEIN NACHLASS

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Bergen, Norway

Since summer 2017, WittFind (<http://wittfind.cis.uni-muenchen.de/>) offers lemmatized search access to the entire Wittgenstein Nachlass as it is available in digital form at the Wittgenstein Archives at the University of Bergen (WAB, <http://wab.uib.no/>). Although the only complete electronic edition of the Nachlass, WAB's *Wittgenstein's Nachlass: The Bergen Electronic Edition* (Oxford University Press, 2000), did offer a number of search and analysis tools which in part remain unmatched, it did not yet include lemmatized search. WittFind represents a significant advance on that and produces, upon entering the lemma of a word (i.e. the dictionary form, "Grundform"), a hit list with all forms of the word that occur in the Nachlass. This means, for example, that one only needs to enter "denken" (or also just "dachte" etc.) in

order to find all occurrences of “denken” and no longer needs to search for “denkt” and “dachten” and “gedacht” and “denken” etc. etc. WiTTFind offers also various other search capabilities based on computational linguistic methods, including syntactic and semantic search in the field of colours and music, or a NLP based method for similarity search. WiTTFind displays the word searched for within the context of the larger remark (“Bemerkung”) and additionally highlights the hit in the corresponding facsimile of the remark. Moreover, WiTTFind is equipped with a separate Facsimile Reader that not only makes paging through Wittgenstein’s Nachlass easy but also additionally contains a function for giving feedback to the editors (be it on the facsimile, the transcription or other parts of the resource). It also comes with a text search and hit-highlighting functionality for the facsimile. WiTTFind is the result of more than five years of close cooperation between WAB and the Centrum für Informations- und Sprachverarbeitung (CIS) at the Ludwig Maximilians Universität München, the former contributing its facsimiles and encoded XML transcriptions of the Wittgenstein Nachlass, the latter providing programming and computational linguistics skills as well as a grammatically encoded digital lexicon of the German language. In our presentation we will demonstrate WiTTFind. We can, upon request, also discuss aspects of the cooperation between WAB and CIS incl. the role of open access and linked data policies, aspects of communication between philosophers, philologists and programmers, work flows as also just the technical infrastructure – which may be useful for anyone interested in starting a digitally mediated (philosophical) cultural heritage collaboration.

RULES, CONSTITUTION, AND CIRCULARITY

Felix Hagenström

Southampton, UK

This paper discusses two of Martin Gustafsson’s contentions in *Wittgenstein, Language, and Chess* (2017): first, that a circularity puzzle arises from the constitutive aspect of rules (the idea that rules constitute expressions and chess pieces), and second, that the analogy between language and chess breaks down as a result of respective differences regarding that aspect. Section 1 sketches rules and the possibility of abstraction from physical features as two elements of the language/chess analogy. Section 2 presents the circularity puzzle and Gustafsson’s suggestion that there is a solution to this in the chess but not in the language case; it then explains why he thinks that this asymmetry leads to breakdown of the language/chess analogy. Section 3 then challenges both Gustafsson’s suggested solution and his characterisation of the breakdown. I conclude that, from a Wittgensteinian perspective, a more fruitful approach to the constitutive aspect of rules and the analogy can be achieved by focusing on our actual practices of speaking and playing chess.

ON THE CONCEPT OF LOGICAL CONSEQUENCE

Volker Halbach

Oxford, UK

The usual model-theoretic account of logical consequence suffers from various shortcomings. In particular, on the model-theoretic account it is not obvious why logical consequence is truth preserving, although truth preservation is presumably the most fundamental feature of logical consequence. In the talk I develop a substitutional account of logical consequence and

demonstrate how it can overcome the shortcomings of the model-theoretic and other accounts.

Roughly, a substitution instance of a sentence is defined as the result of uniformly substituting nonlogical expressions in the sentence with expressions of the same grammatical category and possibly relativizing quantifiers. In particular, predicate symbols can be replaced with formulae possibly containing additional free variables. A sentence is defined to be logically true iff all its substitution instances are satisfied by all variable assignments. Logical consequence is defined analogously. Satisfaction is taken to be a primitive axiomatized notion.

The substitutional definition of logical consequence slots nicely into the place of the elusive notion of informal validity in Kreisel’s Squeezing Argument. Moreover, arguments from logical consequence have been used to argue that absolutely unrestricted quantification is not possible. I show that under the substitutional approach the worries about absolutely unrestricted quantification disappear.

GÖDEL’S EARLY IMPRESSIONS OF INTUITIONISM

Maria Hämeen-Anttila

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In the early 1930s, Kurt Gödel made several contributions to intuitionistic logic. He also examined the question of constructivity of intuitionistic logic, and his critique of intuitionism and the proof interpretation of intuitionistic logic is usually known from his 1958 article published in the journal *Dialectica*. This paper presents a short historical survey of the early development of Gödel’s views on intuitionism based on his publications and shorthand manuscripts. Gödel’s impressions were shaped partly by his readings, which, interestingly, did not include Brouwer, and partly from his own results on intuitionistic logic. The key element in the evolution Gödel’s view is his growing dissatisfaction with the intuitionistic concept of a proof, which is also present in the later writings. Another line of thought, arising from the early formal works, is the interpretation of negative universal statements in intuitionistic arithmetic as existential, a false accusation which was silently buried later.

NORMATIVITY OF LOGIC: THE CASE HUSSERL

Mirja Hartimo

Jyväskylä, Finland

The paper elaborates Husserl’s formal logic as presented in *Formal and Transcendental Logic* (1929). For Husserl, logic as a combination of a theory of judgment and formal ontology has more to do with mathematics than logic in its usual sense. It describes structures rather than offers a theory of inference. Consequently, its normativity differs from the way normativity of logic is usually discussed. Using the distinction between norms for being and norms for action from von Wright’s *Norm and Action*, the paper argues that the normativity of Husserl’s formal logic is distinctive in that it provides exact sciences norms for being rather than norms for action. Norms of being are ideals, or goals for exact scientists’ activities. For Husserl there may be several different sets of ideal norms governing these disciplines (Husserl discusses two in more detail). The ideal norms give rise to logical principles that pertain to reasoning. The normativity of logical principles in turn belongs to norms for action, which are easier to compare to, e.g., Frege’s view of logic. The paper discusses them as providing prescriptive and possibly constitutive norms for reasoning. Their scope is dependent

on the ideal norms. The logical principles thus should not be assumed to be universal but their scope is a matter for transcendental scrutiny.

INTERPRETING TRACTARIAN FACTS

Adam Harwood

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A correct analysis of *Tatsache* in the *Tractatus* is essential if we are to understand it. Such an analysis has been elusive. This paper attempts to go some way to correctly interpreting Wittgenstein's notion of *Tatsache*, and to disambiguate this notion from Russellian facts. *Tatsachen* have often been understood as synonymous with complexes – material entities composed of constituents. This is not how Wittgenstein describes *Tatsache* in the *Tractatus*. The most important contention that I wish to defend is that *Tatsachen* are not composed of anything at all.

This argument has some interesting consequences. Firstly, Wittgenstein's discussion of *Tatsache* in the *Philosophical Remarks* turns out to be entirely consistent with the *Tractatus*, even though it has been argued that he intended to criticise the *Tractatus* on this score. Here we have, perhaps, an instance of Wittgenstein's unreliability as a historian of his own ideas. Secondly, this argument shows the inadequacy of the Ramsey-Ogden translation of *Sachverhalt* as "atomic fact". My argument shows that *Sachverhalte* are different in kind to *Tatsachen*, something this translation precludes. This fame of this translation may have had some part to play in the mistaken belief that *Tatsachen* and *Sachverhalte* are similar in kind, and tangentially, that *Tatsachen* must be composed of parts.

RECONCILING THE NORMATIVE AND THE CONSTITUTIVE NATURE OF LOGICAL RULES: A KANTIAN APPROACH

Jonas Held

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As normative rules, the rules of logic prescribe how we should think, i.e. how to think correctly. This implies that it is possible to think incorrectly, i.e. to think not in accordance with the rules of logic. But this seems to contradict the constitutive nature of these rules. We only think at all if we think according to these rules. I believe that Kant is able to reconcile the constitutive and normative nature of logical rules. Central to this reconciliation is his notion of a capacity. Logical rules are constitutive in the sense that they determine our logical or rational capacity. We only think, i.e. judge and infer, if we actualize our rational capacity to think. But because a capacity can be actualized better or worse, there is room for failure and therefore for the normative role of logical rules. This interpretation will also help us to gain a better understanding of the very nature of logical rules and their relation to thinking in general.

WITTGENSTEIN: INCOMPLETE PICTURES

Richard Heinrich

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Wittgenstein, when he distanced himself from some of the positions held in the *Tractatus*, did not initially give up the picture theory, neither did he give up the concept of elementary propositions as such.

Instead, what he abandoned first was the (atomistic) understanding of elementary propositions as logically

independent and, for a short period, he considered ways to adapt the notion of propositions as pictures to a non-atomistic view of elementary propositions.

The peculiar concept of an incomplete picture is the label attached to one such attempt. It is not primarily meant to single out a special kind of picture, but rather to illustrate what he wants to say about a special kind of proposition: "I think that there is a kind of proposition of which I used to have no idea and which corresponds roughly to what I want to call an incomplete picture" (*Ludwig Wittgenstein and the Vienna Circle*, Oxford 1979, 39).

He focuses on the issue of generality, taking up remark 5.523 of the *Tractatus* ("The generality symbol occurs as an argument") and transforming it into the idea of an elementary proposition (i.e. one containing no logical constants) with free variables.

Topics to be referred to by the way: Placement of this complex of ideas within the broader development of Wittgenstein's philosophy until the mid-thirties (where he still speaks of propositions as pictures); and its possible relation to Bertrand Russell's pre-1905 conception of indeterminate objects.

WITTGENSTEIN'S „SPENGLEREI“

Robert Hofstetter

Wien, Österreich

Vor hundert Jahren erschien der 1. Band von Oswald Spenglers *Untergang des Abendlands*. Wittgenstein hat dieses Werk genau gelesen. Viele Elemente der dort vorgetragenen Zivilisationskritik werden von ihm geteilt. So verabscheut auch er die urbane Lebensform, klagt über den Verfall der Kunst und unterzieht das Fortschrittsmodell einer scharfen Kritik. Mit Blick auf Spengler konstatiert er das Anderssein des Jüdischen. Abschließend wird untersucht, wie all dies mit seinen linken Idealen vereinbar ist.

A TRIANGLE ON TRUTH: LACAN, BADIOU, WITTGENSTEIN

Herbert Hrachovec

Vienna, Austria

1970 Jacques Lacan, in his Seminar XVII: "The Other Side of Psychoanalysis", offers some comments on Ludwig Wittgenstein. Alain Badiou, in a lecture on Wittgenstein as an anti-philosopher, refers to Lacan's considerations (1993). Both theoreticians deal with Wittgenstein's *Tractatus*, which, consequently, serves as target point of a triangulation. Within this conceptual infrastructure an exchange of quotes, explanations and insinuations is taking place. The present contribution proceeds in three steps. (1) Badiou's treatment of Lacan's remarks on Wittgenstein is outlined. (2) This axis of francophone readings of the *Tractatus* is compared to the conventional understanding of this book in German and Anglo-American scholarship. Significant differences become apparent. (3) What, if anything, can be learned from these discrepancies.

WITTGENSTEIN ON THE "DISAPPEARANCE" OF PHILOSOPHICAL PROBLEMS

Liam Hughes

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I examine a tension in Wittgenstein's conception of philosophy; a radical desire to get rid of philosophical problems and a more piecemeal approach. Put simply,

Wittgenstein believes that philosophical problems generally arise because of our failure to understand the grammar of our language. Thus, the task of philosophy is to bring to light what may be concealed or obscured by ordinary or philosophical usage, with the aim of making the problems disappear. However, an examination of problems in ethics/aesthetics casts doubt on taking the notion of "problems disappearing" at face value. Finally, I judge the extent to which the disappearance of philosophical problems is either feasible or desirable.

REVERSING THE CONSEQUENCE ARGUMENT

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In this paper I evaluate van Inwagen's famous Consequence Argument, as presented in *An Essay on Free Will*. The grounds for the incompatibility of freewill and determinism, as argued by van Inwagen, is dependent on our actions being logical consequences of events outside of our control. Particularly, his arguments depend upon, in one guise or another, the transference of the modal property of not being possibly rendered false through the logical consequence relation, i.e. the β -principle. I argue that, due to the symmetric nature of determinism, van Inwagen is exposed to what I call "reversibility arguments" in the literature. Such arguments reverse the β -principle and start from our apparent control over our own actions to our control over the initial conditions. Since van Inwagen does not endorse a particular theory of laws or logical consequence, he is open to such counterarguments. The plausibility of such reversibility arguments depends on what would be called a Wittgensteinian conception of logical consequence. In the *Remarks on the Foundations of Mathematics*, one of Wittgenstein's main concerns is the normativity of logical inference i.e. proof. Such concerns with normativity and rule-following are generally a feature of his later philosophy. In the *Remarks* Wittgenstein resists a conception of logical deduction which places the source of normativity outside of human practice.

WITTGENSTEIN ON IRRATIONAL NUMBERS AND THE INFINITE

Shier Ju & Yikai Ma

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In his middle period, Wittgenstein spends a lot of time discussing the topic of irrational numbers (real numbers). This is a central part of his philosophical interpretation of mathematics, especially, of his opinion on the infinite. On the one hand, just as what he says in *Philosophical Grammar*, "The confusion in the concept of the 'actual infinite' arises from the unclear concept of irrational number." (PG 1974: 471) On the other hand, when criticizing set theory, his accounts of irrational numbers and the infinite can be found almost everywhere in context (such as *Philosophical Remarks* 1975: §173; §181; PG 1974: 460–470). This article is a preliminary to illustrating Wittgenstein's opinion on the infinite in terms of his discussion of irrational numbers (real numbers).

ON EMPIRICAL PROPOSITIONS, "HARDENED EMPIRICAL PROPOSITIONS", AND RULES

Amadeusz Just

Warsaw, Poland

In his later thought Wittgenstein often discussed pictures, grammatical propositions, ideals, rules etc. It seems that all these notions are something entirely different from *empirical propositions*. However, the alleged difference and what the difference consists in, is openly put under discussion in the *Remarks on the Foundations of Mathematics*.

In my talk I shall be concerned with propositions which, however looking like empirical propositions [*Erfahrungssatz*], have other functions. The relation of these propositions to reality is different from that of ordinary empirical propositions. In the *Remarks on Foundations of Mathematics* Wittgenstein described the propositions that only look like empirical propositions but have other function as "hardened empirical propositions" or rules. The similarities between empirical propositions and "hardened empirical propositions" have a tendency to mislead us both in our everyday use of language and in our investigations.

ONTOLOGY IN TRACTATUS LOGICO – PHILOSOPHICUS: A TOPOLOGICAL APPROACH

Janusz Kaczmarek

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Wittgenstein's (and Russell's) ontology of states of affairs or situations was proposed in his *Tractatus*. In the 80s of the 20th century Polish philosopher, Bogusław Wolniewicz, introduced the so called ontology of situations in which Wittgenstein's ontology is interpreted by lattices of elementary situations. In the paper I will propose some topological tools. So, I define Wittgenstein's topology (in honour of Wittgenstein) and – as a theorem – lattice of situations. I will consider also non-atomistic lattice of situations (prepared by topological approach) to investigate differences between atomistic and non-atomistic approach.

EXISTIEREN ZAHLEN?

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Die in der Überschrift gestellte Frage bezieht sich auf Zahlen, also ein Grundlagenthema der Mathematik. Sie tut das aber unter einer bestimmten Rücksicht, nämlich ihrer Existenz. Damit befinden wir uns im Bereich der Ontologie, insofern es dieser Disziplin eben um die Klärung von Existenzfragen geht. In vorliegendem Beitrag soll es aber nicht um ein Plädoyer für eine mögliche Antwort auf diese Frage gehen. Es geht vielmehr um einen Versuch, die Frage selbst zu verstehen. Was bedeutet es, wenn wir nach der Existenz von Zahlen fragen? Wir befinden uns also genau genommen nicht in der Ontologie, sondern auf der Ebene der Metaontologie, die gerade in den letzten Jahren einen bemerkenswerten Aufschwung erlebt hat. Dieser Aufschwung ist, und deshalb eignet sich Metaontologie besonders für unser Thema, wesentlich bedingt durch die Titelfrage und ihre Brisanz. Damit soll auch das Anliegen des Beitrags umrissen sein: Es geht darum, die aktuelle Metaontologie in Bezug zu setzen zur mathematischen Grundlagendebatte und sie als Gesprächspartnerin in diesem speziellen interdisziplinären Diskurs anzubieten.

THE LATER WITTGENSTEIN ON ONTOLOGY

Gary Kemp

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If you think of Wittgenstein and ontology you think naturally of the early Wittgenstein. The late Wittgenstein was not especially interested in ontology or scientific ontology except perhaps for local matters such as the ontology of colour. But I think the certain famous views of his suggest a certain view, a view is preferable to Quine's of the subject. Early in his career Quine held the straightforward view that the ontological question was the question of what science says there is, given a pristine boiled down rendering of the latest science. The answer is an immanent one: science changes and can be wrong, unlike the absolute or transcendental approach to ontology, according to which the question, if answerable at all, specifies what "really" exists, whatever mere empirical science might say. But later in his career Quine found it necessary to qualify even this answer, with his famous doctrine of ontological relativity: what there is is always a matter of interpretation, of translation, in principle if not in practice without end. But this runs smack up against Quine own "paradox" or relativity, that in the very act of proclaiming relativity one purports to rise above it. Wittgenstein's view that there is no other place than forms of life, and that there must be a form of understanding which is not an interpretation, dis-enshrines semantics and re-elevates ontology — or ontology insofar as later Wittgenstein can find room for such a subject — to its rightful place. In this regard, Wittgenstein is more thoroughly naturalistic than Quine.

DISCOVERING INFORMAL LOGIC IN MODERN STUDIES OF ARGUMENTATION

Iryna Khomenko

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In this paper I present my reflections on informal logic. Although it was formed in the late 1970s, consensus on many issues in this field has not been achieved so far. Nowadays it is difficult to identify informal logic as one of the well-defined approaches to argumentation. The aim of this paper is to take a look at the place of informal logic in the state-of-the art studies of argumentation by clarifying its tasks and figuring out the realm to which informal logic belongs.

I emphasize that it is possible to consider informal logic from various perspectives: logical, epistemic and cognitive. The first relates to the normative standards, criteria, and procedures of interpreting real arguments. The second focuses on the problem of real argument assessment. The third concentrates on descriptive study of argument. Informal logic involves appeals not only to the theoretical researches. Empirical studies are important in this area as well.

In the end I conclude that informal logic can be viewed as a normative and descriptive discipline, sitting on the borderline between the interests of logic, epistemology, and cognitive science, with the task to study real arguments through development of certain standards, criteria, and procedures for their interpretation and evaluation.

THREE BASIC TYPES OF LOGIC: DISJUNCTIVE, ADJUNCTIVE AND PREDICATIVE LOGIC

Wolfgang Kienzler

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On systematic grounds, the history of logic can best be divided into three main types of approach, according to two features: i) The use of an exclusive Either-Or ("disjunction"), or of a non-exclusive Or ("adjunction"); and ii) the use of concepts only (term logic), or of objects and concepts (predicate logic). These yield "disjunctive" (Aristotle) and "adjunctive" (Leibniz) term logic, and (adjunctive) "predicate" logic (Frege).

1) The visual paradigm of disjunctive logic is the Tree of Porphyry. Its branches don't overlap: with every new subdivision it is always Either-Or, whether or not the resulting concept has an additional property. The most primitive move is the change from "all" to "none".

2) Adjunctive logic paradigmatically uses Euler diagrams: they introduce independent concepts which may overlap or not. Logic thus becomes the study of (two or more) mutually overlapping circles. Here, the most primitive move is expressed by DeMorgan's Law. The Algebra of Logic tradition swiftly adopted a non-exclusive version of "disjunction" while (confusingly) keeping the name.

3) Frege keeps the non-exclusive Or as being the most suitable for any symbolic logic and introduces the object-concept distinction, together with concepts at several levels.

His most primitive move is the interdefinition of both quantifiers. Against Schröder, who called his logic "Boolean", Frege insists that in the case of his own logic, Euler diagrams are of little use: there is no two-dimensional visualization of "predicate" logic.

The paradigmatic way to illustrate the nature of predicate logic is to use an ordinary sentence like "Socrates is mortal", where "Socrates" stands for an object, and "is mortal", for a first-level concept: $f(a)$. Only "predicate" logic can match the asymmetrical structure of ordinary speech and thereby inaugurate the "logical analysis of language". [Kant, who is often accused of hindering the progress of logic, introduces two types: traditional disjunctive "general logic", and "transcendental logic" which deals with judgments about objects and so foreshadows Frege's invention of predicate logic and the introduction of level-distinctions.]

ON THE REASONABLE EFFECTIVENESS OF MATHEMATICS IN SCIENCE

Peter P. Kirschenmann

Amsterdam, The Netherlands

In 1959, Nobel Prize winner Eugene Wigner delivered a famous lecture, entitled "The Unreasonable Effectiveness of Mathematics in the Natural Sciences", propounding the claim "that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it." More recently, in 2014, Robbert Dijkgraaf, director of the Institute for Advanced Study, Princeton, gave a public lecture, entitled "The Unreasonable Effectiveness of Quantum Physics in Modern Mathematics". Obviously, he argued for some *reverse* effect.

Both physicists mentioned and discussed several examples in support of their claims. Presenting their examples and some of my own, I argue that this effectiveness can never be totally unreasonable. I suggest that there must be reasons for any *particular* successful influence or contribution from one field to the other. And, insofar as there are reasons, the cases concerned should

be intelligible. Yet, noting the reasons in *particular* cases will not distract from our possible existential or cosmic wondering about the whence, wherefore, and whither of nature and mathematics in *general*.

PICTURING COUNTER-INTUITIVE KNOWLEDGE – SEEING ANEW IN WITTGENSTEIN, LEONARDO AND GALILEO

Stephanie Koerner

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This essay explores the implications for fresh hypotheses about taking art and science equally seriously of remarkable parallels between Wittgenstein's investigations of "aspect-seeing" (Day and Krebs 2010); and efforts to rethink the histories of art and science from perspectives offered by scientific illustrations (Biagre 1996). Emphasis falls upon themes of the historical contingency of *aesthetic experience*, *aspect seeing* and *blindness* and *seeing anew*. Examples come from research on Leonard da Vinci's (1452–1519) innovations in optics and picturing the dynamics of realms that are ordinarily invisible (and/or counter-intuitive); and on the roles of illustrations in how Galileo Galilei (1564–1642) addressed challenges posed for his telescopic astronomy.

SIND RECHNUNGEN PSYCHOLOGISCHE EXPERIMENTE?

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In Wittgensteins spätem Nachdenken über die Philosophie der Mathematik rekurriert die Frage, ob Rechnungen psychologische Experimente sind. Oft wird angenommen, dass Wittgenstein glaubt, dass seine Überzeugung, dass mathematische Sätze grammatische Sätze sind, mit einer positiven Antwort auf die besagte Frage unvereinbar ist oder mindestens in Spannung steht. Ich verteidige dagegen eine Deutung, nach der diese Überzeugung zusammen mit Wittgensteins Auffassung des Regel-Folgens als einer Praxis den Hintergrund bildet, vor dem sich ihm diese Frage in ihrer dringlichsten Form allererst stellt: Der Verwendung von Rechenergebnissen als grammatischen Sätzen ist die Übereinstimmung im Rechenverhalten von Menschen vorausgesetzt. Die Annahme dieser Übereinstimmung, die wir scheinbar treffen, indem wir unsere Rechenergebnisse so verwenden, lässt sich nur empirisch rechtfertigen. Wittgenstein weist diesen Gedanken meiner Deutung nach letzten Endes stets zurück, weil er der Ansicht ist, dass die empirische Feststellung der besagten Übereinstimmung selbst Rechnung involviert. Daher setzt die empirische Feststellung der Übereinstimmung im Rechenverhalten von Menschen schon voraus, was sie feststellt und kann diese Voraussetzung nicht rechtfertigen. Ich schließe meinen Aufsatz mit einem Ausblick auf die metaphilosophischen Implikationen dieser Argumentation.

WITTGENSTEIN AND EVOLUTIONARY DEBUNKING ARGUMENTS

Philipp Kremers

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In the recent metaethical discussion, so-called "evolutionary debunking arguments" have received a lot of attention. Despite the popularity of this type of argument, philosophers which locate themselves in the tradition of Wittgenstein have remained silent on this topic for the

most part. In this essay, I aim to fill this gap and explore how a Wittgensteinian response to evolutionary debunking arguments might look like.

BELNAP'S CRITERIA FOR LOGICAL CONSTANTS AND THE PRINCIPLE OF REFLECTION

Hidenori Kurokawa

Kanazawa, Japan

In this paper, we discuss a problem of what a logical constant is from a proof-theoretic viewpoint. We reconsider Sambin et al.'s principle of reflection as a method of introducing logical constants. We discuss a potential problem of the principle of reflection and propose an alternative formulation of it in terms of nested sequents. We also take what they call "solubility of a definitional equation" in this framework to be a positive criterion of logical constant-hood. Based on this approach, we argue that nested sequents can be a suitable proof-theoretic framework in which we can show that Belnap's criteria for logical constants, conservativeness and uniqueness, are satisfied. Consequently, this gives a useful material for further discussing the adequacy of Belnap's criteria, although here we hesitate to be philosophically committed to the view that Belnap's criteria are adequate for logical constant-hood.

WHY THERE IS NO PARADOX IN THE TRACTATUS: ON THE LOGICAL FUNCTION OF WITTGENSTEIN'S SENTENCES

Oskari Kuusela

Norwich, UK

In this paper I explain why there is no paradox of nonsensical theses in the *Tractatus*, and what the logical function of its sentences is. Their purpose is to introduce a Fregean/Russellian logical language by means of which Wittgenstein seeks to clarify the principles of logic that the readers already implicitly know by virtue of being language users. Wittgenstein's logical insights are codified into the rules or structure of this language which constitutes the proper expression for his logical views, not any presumed Tractarian theses. After Wittgenstein's sentences have done their introductory work, they can be discarded and the ladder thrown away. This interpretation explains how Wittgenstein can simultaneously reject philosophical theses, including ineffable ones, and make a positive contribution to logic, the philosophy thereof, and philosophical methodology. By explaining this, the proposed reading solves certain key problems discussed in recent debates on *Tractatus*-interpretation.

ZUR MÖGLICHKEIT EINER BEDEUTUNGSTHEORETISCHEN AUFLÖSUNG DES EPISTEMISCHEN RELATIVISMUS

Konstanty Kuzma

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Gemäß der sogenannten „hinge epistemology“ ist unser Wissen fundiert durch unhinterfragbare Urteile und Überzeugungen. Wissen ist laut dieser Position nur deshalb möglich, weil es gewisse Dinge gibt, die wir als sicher behandeln, obwohl sie nicht eigens gerechtfertigt sind. Dieses Beruhen unserer epistemischen Praxis auf vorausgesetzten Sätzen scheint jedoch die Möglichkeit

eines epistemischen Relativismus zu eröffnen. Denn ist die Frage, was als Wissen gilt, abhängig davon, was als sicher behandelt wird, ist auch denkbar, dass in einer alternativen epistemischen Praxis entgegengesetzte Überzeugungen und Urteile als sicher behandelt würden und dementsprechend auch entgegengesetzte Sätze gewusst werden könnten. Es wäre also prinzipiell möglich, in einer epistemischen Praxis zu wissen, dass X, und in einer alternativen epistemischen Praxis zu wissen, dass $\neg X$. Diese Bedrohung eines epistemischen Relativismus soll unter Berücksichtigung von Wittgensteins *Über Gewißheit* bedeutungstheoretisch aufgelöst werden, indem die bedeutungstiftende Rolle von fundamentalen Urteilen und Überzeugungen herausgearbeitet wird, auf die Baker/Hacker und Forster hingewiesen haben.

WITTGENSTEIN'S CONJECTURE

Timm Lampert

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In two letters to Russell from 1913, Wittgenstein conjectured that first-order logic is decidable. His conjecture was based on his conviction that a decision procedure amounts to an equivalence transformation that converts initial formulas into ideal symbols of a proper notation that decide the logical properties of the initial formulas. According to Wittgenstein, logical properties are formal properties, which are decidable on the basis of pure manipulations of symbols. This understanding of logical properties (such as "logical truth" and "logical falsehood") is independent of and prior to any interpretation or application of logic. Wittgenstein's conception of logic is incompatible with the undecidability proof of Church and Turing from 1936. Thus, Wittgenstein's conjecture and his understanding of logic appear to be refuted. This paper argues (i) that Wittgenstein never withdrew his conjecture and (ii) that he was right in not doing so.

PICTURES IN WITTGENSTEIN'S TREATMENT OF DISAGREEMENT IN THE SO-CALLED LECTURES ON RELIGIOUS BELIEF

Victoria Lavorerio

Vienna, Austria

My aim in this presentation is to show that the religious disagreement explored in the so-called *Lectures on Religious Belief* (Wittgenstein, 1967) about the Last Judgement is rooted in the later Wittgenstein's notion of pictures. I will do this by pointing to three characteristic features of that disagreement and argue that explanation of these revolve around which pictures we use and how. The features are: impossibility of contradicting the other side, breakdown in understanding, and perplexity towards the other's position. The sentence "there will be a Last Judgement" cannot be grasped independently of how it is used; it is not a super-picture. Instead, just like pictures, expressions need methods of projection to say something. Failing to grasp the methods of projection of a sentence entails inability to understand and contradict it, making the interlocutor feel lost and baffled.

MODEL-THEORETIC SEMANTICS FOR MERE EXPRESSIVE DEVICES

Hannes Leitgeb

Munich, Germany

In the first part of this talk I will develop a general model-theoretic semantics for *mere expressive devices*: linguistic

expressions that contribute to linguistic ways of expressing propositions but which do not themselves have a representing function. (Logical operators constitute paradigm case examples of such mere expressive devices.) The semantics will be based on a definition of truth for propositions or sets of models (rather than for sentences) to the effect that truth consists in the existence of a particular kind of structure-preserving map between mathematical models and an intended interpretation. In the second part of the talk, I will use the semantics to throw some new light on various important philosophical debates, such as on the methodology of stipulative definitions, the viability of the logical reconstruction of analyticity, deflationism about truth, a Neo-Carnapian understanding of modal expressions in metaphysics, and the metaphysical status of logic and metaphysics.

WAS MAN AUS EINFLÜSSEN MACHEN KANN – HANS HAHNS ADAPTIERUNG VON RUSSELLS LOGIZISMUS UND WITTGENSTEINS NOMINALISMUS

Alexander Linsbichler

Wien, Österreich

Im Rahmen dieses Beitrags beleuchten wir, wie Hans Hahn die Einflüsse Russells und Wittgensteins adaptiert, um seine individuelle Variante von Logizismus zu entwickeln. Von Bedeutung ist in diesem Zusammenhang Hahns unterschätzte Vorreiterrolle in der Entwicklung des logischen Toleranzprinzips und des logischen Pluralismus, also der Loslösung von der Vorstellung einer einzigen „korrekten“ Logik. Die detaillierten Arbeiten Uebels („Learning Logical Tolerance“, *History and Philosophy of Logic*, 2007, 175–209; „Carnap's Logical Syntax in the Context of the Vienna Circle“, P. Wagnter (Hg.): *Carnap's Logical Syntax of Language*, Palgrave Macmillan, 2009, 57–78) zur Entwicklung des logischen Pluralismus im Wiener Kreis verdienen zumindest zwei punktuelle Ergänzungen: Erstens untermauern die in diesem Kontext bisher unberücksichtigten Protokolle des Wiener Kreises sowie Vorlesungsmitschriften Hermann Brochs die von Uebel im Gegensatz zu Goldfarb („The Philosophy of Mathematics in Early Positivism“, R. Griere; A. Richardson (Hg.): *Origins of Logical Empiricism*, University of Minnesota Press, 1996) beschriebene Distanzierung Hahns von Wittgenstein. Zweitens relativieren Hahns und Mengers Arbeiten zu *Occams Rasiermesser* Uebels Einschätzung, Mengers Beitrag zum Toleranzprinzip sei „wenig philosophisch“.

WITTGENSTEIN, PRIEST'S DIALETHEISM AND CONTRADICTION

Silvia Locatelli

Padua, Italy

The aim of the following paper is to analyse Wittgenstein's later reflections about contradiction, especially focusing on his *Remarks on the Foundations of Mathematics*, lining them up with his *Lectures* held in Cambridge in 1939 and *On Certainty*, which contains thoughts that belong to the last year and a half of Wittgenstein's life. More room will be dedicated to two passages, found in the *Remarks*, namely on §79 in section II and on §8 in section V, sufficiently to figure out if contradictions on Wittgenstein's view are to be considered really acceptable or not. In view of these passages I want to sustain that for Wittgenstein the occurrence of a contradiction is not always a problem to avoid, but it could be a natural and appropriate way to describe the surrounding world. To

conclude, a brief parallel between Wittgenstein and Dialetheism will be proposed, more specifically with respect to a passage from *In Contradiction* by G. Priest, to show how Priest's particular position about the contradiction's ontologic foundation could represent a key to interpretation for the *Remarks'* passages analysed.

TRANSLATIONS BETWEEN LOGICS

Itala Maria Loffredo D'Ottaviano

Campinas, Brazil

For several years the interrelations between logics have been studied by analysing translations between them.

In 1999, da Silva, D'Ottaviano and Sette proposed a very general definition for the concept of translation between logics; logics being characterized as pairs constituted by a set and a consequence operator, and translations between logics being defined as maps that preserve consequence relations.

In 2001, with Feitosa, we introduced the concept of conservative translation and studied the category whose objects are logics, and whose morphisms are the conservative translations between them. In 2007, Carnielli, Coniglio and D'Ottaviano proposed the concept of contextual translation in order to have a stricter notion of translation and to solve questions related to conservative translations. Conservative and contextual translations showed themselves, however, to be independent concepts. Recently, with Almeida and Feitosa, we introduced the concept of abstract contextual translation between logics and proved that this new concept is an intermediate concept, wider than the concepts of conservative and contextual translation. We also studied other, stricter, concepts of translations: the conservative-contextual and the hypercontextual translations.

Providing some brief historical background, I present a general survey of the main questions and problems we have analysed and the results we have obtained.

As well as showing the interrelations between these concepts of translation and the concept of isomorphism between logics, I discuss the interrelations among the distinct categories that are constituted by logics and the special types of translations between them.

INWIEFERN IST DIE FRAGE „INWIEFERN IST DIE LOGIK ETWAS SUBLIMES?“ EINE METAPHILOSOPHISCHE FRAGE?

Linus Lutz

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„Inwiefern ist die Logik etwas Sublimes?“ scheint nicht bloß die Thematik von §89, der mit dieser Frage einsetzt, anzuzeigen, sondern maßgeblich zu sein für eine Reihe darauf folgender Paragraphen der *Philosophischen Untersuchungen (PU)*. Wie in einem ersten Schritt dargestellt wird, lässt sich die Frage nach der Sublimität der Logik genauer als expliziter Ausgangspunkt eines metaphilosophischen Abschnittes der *PU* auffassen. In einem zweiten Schritt werden dann ein alternatives Verständnis des Begriffes der Logik und ein daraus erwachsendes exegetisches Problem herausgestellt. Wenn mit „Logik“ nicht wie zuvor gedacht auf die Philosophie, sondern auf die Logik der Sprache Bezug genommen wird, dann ist unklar, ob und inwiefern sich die Frage nach der Sublimität der Logik noch immer als natürlicher Ausgangspunkt des metaphilosophischen Abschnittes der *PU* auffassen lässt. Der dritte Schritt stellt den Versuch einer Lösung dieses Problems dar. Die Frage nach der Sublimität der Logik betrifft gemäß dem alternativen Begriffsverständnis zwar buchstäblich nicht

die Philosophie, sondern die Sprache. Dennoch lässt sich diese Frage insofern als metaphilosophisch und als Ausgangspunkt des entsprechenden Abschnittes der *PU* deuten, als sie die Sprachauffassung des *Tractatus logico-philosophicus* in einer Weise adressiert, die nur wenig mittelbar auch dessen Philosophieauffassung thematisch werden lässt. Als Ausblick wird angedeutet, dass sich die Frage nach der Sublimität der Logik nicht nur unter Voraussetzung beider möglicher Verständnisse des Begriffes der Logik als metaphilosophisch auffassen lässt, sondern dass zwischen beiden Lesarten der Frage noch weitere Übereinstimmungen bestehen.

A PICTORIAL ASPECT OF MATHEMATICAL NOTATION IN WITTGENSTEIN: PROOFS

Jakub Mácha

Brno, Czech Republic

The core in Wittgenstein's conception of mathematics can be summed up in the motto that "arithmetical rules are statements of internal relations" (*Public and Private Occasions*, Rowman & Littlefield, 2003, 390). I am going to focus on Wittgenstein's insistence on a certain pictorial aspect of mathematical notation, which is, of course, his Tractarian heritage. Mathematical notation must always be capable to depicture a state of affairs. This is true of numbers, but also of mathematical proofs. Numbers and proofs are for Wittgenstein a sort of prototypes of certain activities. Mathematical propositions are statements of internal relations as well. A proof of a mathematical proposition aims to picture or rather lay down its internal relatedness to a system of other mathematical rules. We may say that "the completely analysed mathematical proposition is its own proof." (*Philosophical Remarks*, Blackwell, Oxford, 1975, 192) Proof is so a picture of an experiment, even more "it can be thought of as a cinematographic picture" (*Remarks on the Foundations of Mathematics*, Blackwell, Oxford, 1978, 159).

NEOLOGICIST FOUNDATIONS: INCONSISTENT ABSTRACTION PRINCIPLES AND PART-WHOLE

Paolo Mancosu (joint work with Benjamin Siskind)

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Neologicism emerges in the contemporary debate in philosophy of mathematics with Wright's book *Frege's Conception of Numbers as Objects* (1983). Wright's project was to show the viability of a philosophy of mathematics that could preserve the key tenets of Frege's approach, namely the idea that arithmetical knowledge is analytic. The key result was the detailed reconstruction of how to derive, within second order logic, the basic axioms of second order arithmetic from Hume's Principle

$$(HP) \forall C, D (\#(C) = \#(D) \leftrightarrow C \cong D)$$

(and definitions). This has led to a detailed scrutiny of so-called abstraction principles, of which Basic Law V

$$(BLV) \forall C, D (ext(C) = ext(D) \leftrightarrow \forall x (C(x) \leftrightarrow D(x)))$$

and HP are the two most famous instances. As is well known, Russell proved that BLV is inconsistent. BLV has been the only example of an abstraction principle from (monadic) concepts to objects giving rise to inconsistency, thereby making it appear as a sort of monster in an otherwise regular universe of abstraction principles free from this pathology. We show that BLV is part of a family of inconsistent abstractions. The main result is a theorem to the effect that second-order logic formally refutes the existence of any function F that sends concepts into objects and satisfies a "part-whole" relation. In addition, we

study other properties of abstraction principles that lead to formal refutability in second-order logic.

FOLLOWING A RULE: WAISMANN'S VARIANT

Mathieu Marion (joint work with Mitsuhiro Okada, Tokyo, Japan)

Montréal, Canada

In this paper, we wish to reconstruct on the basis of F. Waismann a *variant* by Waismann of Wittgenstein's rule-following argument (see *The Principles of Linguistic Philosophy*, MacMillan, 1997, 119–124, and "Causality", in B. McGuinness (ed.), *Friedrich Waismann: Causality and Logical Positivism*, Springer, 2011, §§10f.). We contrast its basis, language-game # 62 in Wittgenstein's *Brown Book*, with Kripke, the pupil's case (see L. Wittgenstein, *The Blue and Brown Books*, Blackwell, Oxford, 1969, 112; *Philosophical Investigations*, §§143, 152, 179, & 185; S. Kripke, *Wittgenstein on Rules and Private Language*, Harvard University Press, 1982). It involves X writing down the initial segment of a series, and Y trying to guess which rule has been followed. According to Waismann, Y can only come up with "hypotheses" that are *causal*, while X's claim to have followed a given function is a *reason* justifying her actions, and if why-questions are then raised, this "chain of reasons" will eventually come to an end. We then focus on parallels with Lewis Carroll's paradox of inference in the case of "basic rule-following" such as applying Modus Ponens, showing that this variant helps us handling such cases, while avoiding difficulties such as the infinite regress in the "chain of reasons" (see C. Wright, "Rule-Following without Reasons: Wittgenstein's Quietism and the Constitutive Question", *Ratio*, Vol. 20, 2007, 481–502).

ECHOES OF WITTGENSTEIN IN TOULMIN'S MODEL OF ARGUMENT: AN INTERPRETATION OF TOULMIN'S MODEL FROM THE POINT OF VIEW OF THE TRACTATUS

Kateřina Merglová

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It is obvious that ideas of teachers can have a great impact on their students' opinions. My aim here is to try to uncover if Ludwig Wittgenstein had an impact on Stephen Toulmin not just as a teacher but as a philosopher. For this purpose, I first introduce Toulmin's model of argument in its later modification. For better understanding, I illustrate this model with an example of an argument from science. Then I present selected ideas of the early Wittgenstein from *Tractatus Logico-Philosophicus* and use these ideas to interpret Toulmin's model. I point to one difference between Wittgenstein and Toulmin which is in my opinion the most important. At the end, I refer to a resemblance between Toulmin's opinions about formal logic and the development of Wittgenstein's thoughts about language.

MENGEN – EINE KLEINE ZUSAMMENFASSUNG (UND WARUM DIE MATHEMATIK KEINER GRUNDLEGUNG BEDARF)

Gabriele M. Mras

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Der Artikel will zeigen, dass der im Mengenbegriff gegebene Standpunkt des Zusammenfassens und

Vereinheitlichens eine dem mathematischen Denken adäquate Sichtweise darstellt, aber dass er, bzw. die Mengenlehre, keinesfalls, wie gerne behauptet, die Grundlage der Mathematik sein kann. Anhand der Überlegungen Freges zur Zahl wird der Nachweis erbracht, dass die Auffassung der Zahl als Menge ganz grundsätzlich scheitern muss – und nicht erst in ihrer Konsequenz bzw. aufgrund der mengentheoretischen Antinomien. Freges (inkonsistente) Ausführungen zur Beziehung von Zahl und Zeichen und seine Missverständnisse bezüglich der Frage, wem die Zahl als Eigenschaft zukomme, werden ebenso aufgeklärt wie das Verhältnis zwischen den Kategorien „Menge – Element“ und „Ganzes – Teil“. Und es ergibt sich dabei, dass die Mathematik – ganz im Wittgensteinschen Sinne – keiner Grundlegung bedarf.

REDUCTIONS OF MATHEMATICS: FOUNDATION OR HORIZON?

Felix Mühlhölzer

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The usual reductions of large parts of mathematics to much more restricted parts, with the reduction to set theory as a sort of paradigm, are virtually uncontroversial from a purely mathematical point of view. But what is their point? According to the standard answer, they are important because they provide *foundations* for mathematics. What that precisely means, however, can be explained and also be criticised in quite different ways. There is a Wittgensteinian way of criticism that proves to be particularly instructive and that is summed up in the following beautiful passage in §16 of *Remarks on the Foundations of Mathematics* VII: "The *mathematical* problems of the so-called foundations are not more at the basis of mathematics for us than the painted rock is the support of a painted castle."

If this criticism is correct (as I will argue), what then can be the point of the reductions? There is a good answer given by Bourbaki: such a reduction provides an *horizon* for mathematics. This is a totally different idea from the idea of a foundation! The horizon of mathematics is understood as a perfect formalization that lies *in front* of us and that guides us, but it is not *beneath* us like a foundation, i.e. a sort of rock that supports the edifice of mathematics. The Bourbakist Claude Chevalley criticised the idea of an "horizon", but it can very well be defended and then proves to be compatible with Wittgenstein's criticism of "foundations".

ŁUKASIEWICZ, OCKHAM'S RAZOR: OPERATIVER NON-DUALISMUS

Wolf Dietrich Nagl

Kirchheim bei München, Deutschland

Die meisten Lehrbücher der Logik widmen den Meta-Axiomen von Łukasiewicz nur einen Nebensatz, der Experimentalphysiker Ernst Mach weist in ((1903) *Populär-Wissenschaftliche Vorlesungen*) auf Ockhams Rasiermesser hin. In seiner Tradition strukturieren hier die Formale Aussagenlogik (Cameron (1999) *Sets, Logic and Categories*) und Ockhams Rasiermesser (Duns Scotus (1987) *Abhandlung über das erste Prinzip*) die Voraussage und die anschließende Realisierung von Sinneseindrücken in Naturwissenschaft und Wirtschaft. Das erfolgt mit abstrakten Modellen und in Einzelschritten mit erprobten Messgeräten und Aktoren. Gesteuert wird das von simulierten Universalien, die im Bewusstsein über Objekte und Dokumente in der körperlichen Umwelt subjektiv gestaltet werden, also nicht aus dem dualen Jenseits

stammen (Mitterer (1993) *Das Jenseits der Philosophie*). Ihre Sinneseindrücke unterstützen auch das subjektive Gedächtnis bei der zeitlichen und interpersonellen Stabilität. Für die Realisierung konkreter Voraussagen werden simulierte Universalien ausgewählt und dann fundamentalistisch abgearbeitet. Viele von Historikern und Archäologen untersuchte Funde haben ursprünglich zur Simulation von Universalien beigetragen. Beispiele dafür gibt es in der modernen Naturwissenschaft seit einigen Jahrhunderten und in Wirtschaft und öffentlicher Verwaltung seit einigen Jahrtausenden. Die intuitiven Schnittstellen von Mixed und Augmented Reality sowie auf Big Data und Deep Machine Learning beruhende Matrizenrechnungen erschließen nun diesem Wissen weitere Einsatzgebiete.

WITTGENSTEIN AND GRISS ON NEGATION AND FALSITY IN MATHEMATICS

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The present work presents some connections between the negationless intuitionistic mathematics, developed by the Dutch mathematician George Griss between 1946 and 1951, and Wittgenstein's reflections about negation and falsity in mathematics at the beginning of his middle period (particularly in *Philosophical Remarks*). We shall see two main points of contact between these authors. The first is the suspicion about unrealizable suppositions in mathematics (used in indirect reasoning to prove the opposite of a certain assumption). In both cases, the suspicion arises from a modal interpretation of mathematical statements, according to which false propositions in mathematics are absurdities. Another point of contact between Wittgenstein and Griss is the idea that false propositions are not needed for the construction of the mathematical edifice. The conclusion that both authors draw from this idea, however, is not that negation in mathematics is entirely superfluous, but rather that negation only becomes useful in mathematics when connected to generality.

WITTGENSTEIN ON THE SCANDAL OF DEDUCTION

Antonio Negro

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The goal of this paper is to sketch a take on the scandal of deduction that is in agreement with the *Tractatus*. Traditionally, it is maintained that (i): the conclusion of a deductive inference is contained in its premises. But then, the scandal goes, (ii): a deductive inference is useless. The containment account of logical consequence to which (i) refers is propounded in the *Tractatus* in terms of sense containment; and, in some later comments, Wittgenstein stressed that sense containment is not to be conceived in psychological terms, thus providing a way to oppose (ii). However, these later comments appear in tension with the Tractarian notion of sense as correlated to understanding. My interest is to show how one can retain (i) and resist (ii) by defending a Tractarian proposal. I will focus on two perspectives on the notion of sense, both taken in consideration in the *Tractatus*: a negative and a positive perspective. Sense containment (hence, (i)) holds if rephrased in terms of logical content, namely by conceiving of the sense of a sentence in terms of how things do not stand if it is true (as *per* the negative perspective). Sense containment does not hold if rephrased in terms of representational content, namely by conceiving of the sense of a sentence in terms of how

things stand if it is true (as *per* the positive perspective). Since understanding is correlated to the positive perspective, one gets an explanation of the usefulness of deductive inferences and resists (ii).

EPISTEMIC LOGIC AND HOW IT CAN EXPLAIN OUR MATHEMATICAL KNOWLEDGE

Dan Neshier

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What is logic and what is its role in human affairs is the basic epistemological question. Epistemic Logic is the basic science representing our confrontation in reality by proving the truth that we actually represent it. The formal systems are just a closed game of argumentations that assumes the truth and the falsity of the initial axiomatic propositions, and by just assuming the validity of the inferences, we reach their conclusions. The difference between formal systems and realist theories lies in their different proof-conditions when formal systems are hermetically closed games under their fixed axioms which cannot be proved true, when their formal rules of inference cannot evaluate the truth of their theorematic conclusions to reality. Hence, axiomatic formal systems are complete and isolated from reality while the realistic theories are Gödelian incomplete but can be proved true relative to their proof-conditions. However, if mathematics is to be theoretical science it cannot be pure axiomatic closed systems isolated from reality, but an empirical science, and thus mathematicians can avoid the ambiguity, contradictions, and paradoxes.

THE TRACTARIAN ACCOUNT OF INFERENCE

Gilad Nir

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In the *Tractatus Logico-Philosophicus* Wittgenstein argues that inferential justification does not require an appeal to logical rules. Instead, it consists entirely in having a proper understanding of the premises and conclusion. This account of inference is grounded in Wittgenstein's holistic conception of understanding, according to which propositions form a network of "internal relations" which inform our capacity to use them. The account contrasts with the standard and still widespread view that the justificatory nature of inference depends on there being, apart from premises and conclusions, a separate act or state of evaluating the goodness of the inferential connection. Wittgenstein's account deserves the attention of contemporary theoreticians of inference, since it overcomes two main challenges that the standard view faces: the worry that the evaluative element of inference lacks causal efficacy (giving rise to regress), and the worry that the causal aspect of inference is dissociated from its evaluative aspect (rendering inference indistinguishable from mere associations of thought).

WITTGENSTEIN'S QUASI-INTUITIONISM

Luca Oliva

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Is Wittgenstein an intuitionist? It's unclear whether he rejects or emends Brouwer. His logical atomism relies on correspondence, while his mathematical constructivism doesn't. Scholars are divided. Following Russell, Wittgenstein endorses a *fact-based version of correspondence*. The Aristotelian truth-definition, which

can be reduced to “ x is true iff x corresponds to some fact”, is restricted to a subclass of truth-bearers, namely *elementary* propositions whose truth consists in their correspondence to state of affairs. On the other hand, Wittgenstein dismisses *the law of excluded middle*, “ $(x)Fx \vee (\exists x)\sim Fx$ ”. “ $P \vee \sim P$ ”, for instance, doesn’t hold for infinite sequences since it doesn’t tell whether the pattern φ (any particular arrangements of digits) occurs in the infinite expansion π or not. In this paper, I shall examine the tension between *realism* and *intuitionism* in Wittgenstein’s philosophy, where his relying on correspondence seems to conflict with his rejection of the law of excluded middle. I shall finally accommodate the two within a single, coherent view on mathematics that might be seen as *quasi-intuitionism*, where mathematics is reduced to mental manipulations of signs (consistently with any degree of constructivism) that yet resist any mental dependency.

FINDING HIMSELF IN THE NEIGHBORHOOD OF WITTGENSTEIN’S THINKING: WITTGENSTEIN’S INFLUENCE ON VON WRIGHT AND VON WRIGHT’S UNDERSTANDING OF WITTGENSTEIN

Bernt Österman

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In my paper I will discuss the influence Ludwig Wittgenstein had on the philosophical thinking of one of his three literary executors and heirs, the Finnish philosopher Georg Henrik von Wright (1916–2003). It is shown how Wittgenstein’s influence went through several stages, with von Wright’s struggle for independence as a central theme. Eventually, in the 1970s, von Wright became aware of a “latent influence” that had brought his own thinking “in the neighborhood of Wittgenstein’s”, which he started to exploit. I will argue that this is the time when we are starting to see traces of von Wright’s own philosophy in his understanding of Wittgenstein. Finally, I will ask to what extent his new understanding(s) of Wittgenstein also affected his editorial work on Wittgenstein’s *Nachlass*. I will use *Culture and Value (Vermischte Bemerkungen)* as an example, the compilation of general cultural remarks by Wittgenstein von Wright first published in 1977. I will argue that von Wright, actually, was advancing a conception of Wittgenstein as a “philosophical feedbacker”, reacting to the illness of our time – a role von Wright himself was striving to take during these years.

IS LANGUAGE SOCIALLY CONSTRUCTED? A STUDY IN WITTGENSTEIN’S PHILOSOPHY

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Language is not psychologically arrived at by abstraction, nor is it a genetically transmitted medium of learning and communication, much less a matter of subjective experience of the speaker. Language is a inter-subjective social phenomenon. The primary units of any language, namely, the words and sentences are used and understood in the shared community of speakers. Thus, language and meaning are socially constructed.

The focus of this paper will be on the later philosophy of Wittgenstein who rejects the privacy of language and makes instead use the criterion of the meaningfulness of words and sentences. The aim of this paper is to locate use in the social reality of the individual that is a part of the linguistic community. Owing to the dynamism inherent in the social use of language, it goes on incorporating ever-

newer words into its vocabulary such that the older words are either discarded or modified to suit the present needs.

A NOTE ON THE AVOIDANCE APPROACH TOWARD DOXASTIC PARADOX

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Given some doxastic logic, Hintikka-paradoxical sentences are those sentences that are satisfiable but refutable if believed. The standard examples of doxastic paradoxes are the Moore and Buridan-Gödel sentences. Considering the standard logics for modeling doxastic agents, should we prefer logics in which the believed versions of these sentences are refutable? One might answer affirmatively assuming that to refute the believed sentence is to avoid paradox. One motivation for this approach is the thought that these sentences are intuitively unbelievable for rational agents. In this note, we examine the extent to which this intuition motivates the preference for logics in which the believed versions of these sentences are refutable. We argue that such a motivation is weak by providing a theory (namely, the recently introduced Doxastic Arithmetic (DA)) within which a rational agent believes a Buridan-Gödel sentence.

THE LATER WITTGENSTEIN ON THE SEMANTICS OF CONTRADICTIONS

Alessio Persichetti

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This paper portrays later Wittgenstein’s idea of what means for a contradiction to have sense, through the *Lectures on the Foundations of Mathematics*, plus the *Remarks on the Foundations of Mathematics*. This comes in contrast with a certain tradition from Hilbert to Frege and Russell, which deems a contradiction as a fatal flaw for a formal system. I will argue that for Wittgenstein a contradiction isn’t a problem: it represents an issue as long as we don’t know what to infer precisely from it. Once a meaning is established, then the contradiction becomes a usable expression like many others. Moreover, I will explain why this attitude towards contradictions is rooted in Wittgenstein’s anti-realistic conception of mathematics.

WITTGENSTEIN AND FREGE ON ASSERTION

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In the *Philosophical Investigations*, Wittgenstein famously criticizes Frege’s conception of assertion. “Frege’s opinion that every assertion contains an assumption” (§22) rests on the possibility of parsing every assertoric sentence into two components: one expresses the thing that is asserted, the other expresses *that* it is asserted.

But this possibility does not entail that the “assertion consists of two acts, entertaining and asserting” any more than the possibility of rendering assertions as pairs of questions and affirmative answers entails that they consist of questions.

Frege scholars protest that such criticism is inappropriate, not only because Frege doesn’t speak about assumptions, but also – and crucially – because Wittgenstein fails to address the logical nature of assertion reflected in Frege’s use of the judgment stroke. They appear to read Wittgenstein’s argument by the light of a remark in the *Tractatus* saying that the judgment stroke is “logically meaningless” because it simply indicates that the author

holds the propositions marked with this sign to be true (4.442).

Wittgenstein always considered Frege's assertion sign superfluous. I argue that his criticism of Frege's conception of assertion should not be reduced to the mysterious role of this symbol and shows a deep disagreement about the nature of logic and language.

DIE SPRACHLOGIK VON WITTGENSTEINS TRACTATUS ALS FREIE BOOLESCHE ALGEBRA

Martin Pilch

Wien, Österreich

Der Beitrag untersucht den algebraischen Zusammenhang von Wahrheitsargumenten, Wahrheitsmöglichkeiten und Wahrheitsbedingungen im *Tractatus*. Die in TLP 4.27 und 4.42 angeführten Formeln legen nahe, dass die Sprachlogik des *Tractatus* – zumindest für den Fall, daß die Gesamtzahl der Elementarsätze endlich ist – die Form einer „freien Boolesche Algebra“ hat. Die Elementarsätze übernehmen darin die Funktion der „freien“ Generatoren der Lindenbaumalgebra der (klassischen) Aussagenlogik. Eine Konsequenz dieser Deutung ist, daß die Annahme der Eindeutigkeit der Satzanalyse (TLP 3.25) nicht mit der wahrheitsfunktionalen Konstruktion der Sätze (TLP 5) vereinbar ist.

A PROCEDURAL ACCOUNT OF CARDINAL NUMBERS

Martin Pleitz

Hamburg, Germany

I present a variant of the neo-logicist account of cardinal numbers that is based on Kit Fine's procedural postulationism. This amounts to splitting up the relevant abstraction principle (Hume's principle) into a hypothetical identity criterion for cardinal numbers, which is purely conceptual, and existential claims about cardinal numbers, which are imperatival. This approach solves (what I call) the Anselm problem for neo-logicism, which arises because it aims to show that arithmetic including all its existence claims about natural numbers is analytic although no existence claim can be analytic. I will also remark on how the procedural account of cardinal numbers solves the Caesar problem and the Bad Company problem, and remark on how it relates to recent work on dynamic abstraction by Øystein Linnebo.

PROPOSITIONS IN WITTGENSTEIN AND RAMSEY

Michael Potter

Cambridge, UK

According to the *Tractatus* it is impossible to express the internal properties of objects. In his "Critical Notice of L. Wittgenstein's *Tractatus*" (*Mind*, 32, 465–478, 1923) Ramsey suggested that a general account could nonetheless be given of the origin and apparent significance of these properties which has "no mystical implications". I explain how a difference between Ramsey's conception of propositions and Wittgenstein's explains why Ramsey thought this, and I consider the consequences this has for other aspects of the *Tractatus*, in particular the accounts of belief and of the theory of types.

PREDICATION, RELATIONS, PARTICULARS

Cyril Pshenichny, Uwe Wolter & Sergey Dzhura

St. Petersburg, Russia

Bergen, Norway

Donetsk

One of the key ideas of early Wittgenstein and logical positivism, and then of much of the Western philosophy of the last hundred years is that the world can be comprehended via the language used to describe it. Once this language is strict enough, the world becomes comprehensible, structured in our mind and prone to treatment by formal tools that bring new knowledge about it based on that we already have. This has inspired Frege to think about *logicism* as the program of thorough formalization of the mathematics by means of logic (Frege *Grundgesetze der Arithmetik*, 1893/1903), and some pioneers of artificial intelligence went yet further, claiming, e.g., that computer reasoning can endlessly mimic the structure of the world drawing conclusions from our previous knowledge (McCarthy "Programs with Common Sense", 1968). Then, in fact, the only issue that requires attention should be language. However, with time it has become obvious that the language used by modern logic, philosophy and knowledge representation achieves its formality at the expense of poor sensitivity to many relevant features of the world it describes. This paper aims to suggest a possible solution of this problem based on the novel ontological theory, the theory of multitudes.

ON THE VARIETIES OF MORAL CRITICISM

Richard Raatzsch

Wiesbaden, Germany

Moral criticism is not the only kind of criticism, nor is criticism the only moral activity. This seems to indicate that there is something like a general form of criticism, and that moral criticism has this form plus some moral aspect, whatever this may be.

Yet, if this were true, moral criticism could not be essential to morality. So, if moral criticism is to be essential for morality, the notion of criticism can only be (fully) understood if the (sub)notion of moral criticism is already understood.

The idea of a general form of criticism does, however, help us to see why there is something is called moral criticism. Wittgenstein's idea of family resemblance does justice, I propose, to this problem. This paper offers a first sketch of the varieties of moral criticism, based on this idea.

WITTGENSTEIN ON MATHEMATICAL UND RELIGIOUS PROPOSITIONS

Esther Ramharter

Vienna, Austria

At first glance, mathematical and religious propositions seem to be located on diametrically opposed ends of the spectrum of certainty: the former being considered as the very paradigm of certainty, the latter as unsure and arguable.

In the philosophy of Ludwig Wittgenstein, however, these two sorts of propositions, seen from an epistemological point of view, tend to converge. Both can be said to be "hinge propositions" – propositions large parts of our language and beliefs rest on, both are normative.

What thus seemed to be very different at first sight becomes very close from a certain point of view. For both mathematical and religious propositions their normative and foundational status is central, if not to say characteristic.

To get a clearer picture of the similarities and differences between them it is useful, as I will try to show, to distinguish internally different types of each of those kinds of propositions.

WITTGENSTEIN'S CRITIQUE OF MATHEMATICAL FORMALISM

Enakshi Ray Mitra

Delhi, India

The first section of this paper is a preliminary exposition of Formalism as a theory of mathematics – in its term-formalistic and rule-formalistic variations – highlighting their respective lacunas. The second section presents a brief account of D Hilbert's programme of outgrowing these defects of naïve Formalism – mainly with reference to his later theory of Finitism that sought to base mathematics on the pre-conceptual and intuitive character of signs. The main agenda of this paper is carried out in the last section, which attempts to construct a later Wittgensteinian resistance to this pre-conceptual nature of signs, by showing how their putative identities break out into a flow of practices.

UNDERSTANDING WITTGENSTEIN'S WOOD-SELLERS

Štefan Riegelnik

Zurich, Switzerland

In the collection *Remarks on the Foundations of Mathematics* (I, 149) Wittgenstein encourages us to imagine a group of people selling wood at a price relative to the area covered by the pile of wood irrespective of the height of the pile.

In "Wittgenstein and Logical Necessity" Barry Stroud argues that Wittgenstein uses this scenario (and others) to steer between two untenable positions: (i) Frege's Platonism, according to which the "wood-sellers" must be considered insane, and (ii) a version of conventionalism which leaves open the possibility of ways of inferring, counting, and calculating that are different to ours.

At first sight, the behaviour of the wood-sellers seems to be comprehensible. But, as Stroud argues, the more we project our grammatical structures and categorisation into their verbal and non-verbal behaviour, the less intelligible the wood-sellers become. On the one hand, this shows that the way we calculate, infer, count etc. depends on contingent facts, hence Platonism is misguided. On the other hand, since the wood-sellers' alternative ways of calculating, inferring, counting, etc. cannot be made intelligible, conventionalism is to be rejected as well.

In my talk, I discuss Stroud's interpretation of the example and examine the role of *consistency* as a guiding principle for the projection of our grammatical structures into the behaviour of the wood-sellers. I argue that consistency is responsible for the unintelligibility of the wood-sellers. Since we do not succeed in understanding their behaviour, I consider whether we are able to imagine the wood-sellers as humans, and whether we can recognise their sounds as linguistic.

GÖDEL'S "GREAT SERVICE" TO WITTGENSTEIN'S PHILOSOPHIES OF MATHEMATICS

Jurgen Rinkel

Leiden, The Netherlands

Wittgenstein's later philosophy of mathematics can be divided into two periods, dominated by two different conceptions: the calculus conception and the language-game conception. In this paper I will argue that one reason Wittgenstein abandoned the first in favour of the other was Gödel's 1931 proof of the First Incompleteness Theorem. Wittgenstein's remarks about this Theorem as given in *Remarks on the Foundations of Mathematics (RFM)* I.III show him to be confronted with a different situation which was incompatible with the calculus conception and therefore forced him to reject the latter. The replacing language-game conception, on the other hand, was able to accommodate the peculiarities of Gödel's result and is at least hinted to in *RFM* I.III. Connected with this change of conceptions was the deflation of the notion of "mathematical proposition", which reinstated a belief already apparent in the period of the *Tractatus*. Incidentally I will explain why Wittgenstein thought Gödel to have delivered a "great service to the philosophy of mathematics".

BEYOND FOUNDATIONALISM: FREGE AND WITTGENSTEIN ON THE RELATION BETWEEN LOGIC AND MATHEMATICAL PRACTICE

Tabea Rohr

Jena, Germany

During the last decades, scholars working in philosophy of mathematics have increasingly become interested in our mathematical practice. Philosophy of mathematical practice is commonly believed to originate with Lakatos. In this paper however, it will be shown that Frege emphasizes (especially in "Boole's Calculating Logic and the Concept-script") that our mathematical practice cannot be replaced by a mere calculus, but involves creative elements, in particular concept formation. Then the consequences of this insight for Frege's understanding of the analyticity of arithmetic will be discussed. Finally it will be shown that Wittgenstein, when he argues against foundationalism in the *Remarks on the Foundation of Mathematics*, advances considerations closely related to the points Frege argues against Boole – only that Wittgenstein draws the consequences from them more radically.

TRUE IN ABSOLUTELY EVERY CONTEXT

Lorenzo Rossi

Salzburg, Austria

Contextualist theories of truth propose to block semantic antinomies postulating a context shift in the course of paradoxical derivations; a Liar sentence λ equivalent to " λ does not express a true proposition" does not express a true proposition in the initial context of reasoning, but expresses a true one in a richer context (see e.g. C. Parsons, "The Liar Paradox", *Journal of Philosophical Logic*, Vol. 3, No. 4, 1974, 381–412; M. Glanzberg, "A Contextual-Hierarchical Approach to Truth and the Liar Paradox", *Journal of Philosophical Logic*, 33, 2004, 27–88; K. Simmons, *Universality and the Liar*, Cambridge University Press, 1993). However, existing contextualist approaches are incompatible with absolutely unrestricted

quantification. Since λ doesn't express a proposition in the initial context of reasoning but it expresses one in a richer context, and the liar reasoning can be replicated in any given context, contextualist wisdom has it, absolute generality must be given up (Glanzberg, 2004). But rejecting absolute generality comes at a very high price: it forces truths such as "Everything is self-identical" to be less than absolutely general (T. Williamson, "Everything", *Philosophical Perspectives*, Vol. 17, No. 1, 1993, 415–465). In this paper, we propose a way to make the contextualist treatment of the paradoxes compatible with a form of absolute generality. With the help of tools for diagnosing paradoxical sentences including revenge-breeding sentences (L. Rossi, "A Unified Theory of Truth and Paradox", forthcoming), we distinguish between sentences that express a true proposition in absolutely every context (e.g. "everything is self-identical") and sentences that cannot (e.g. Liar sentences), including the resulting revenge paradoxes in the latter category. The context-shifting semantics is only applied to the paradox-prone fragment of the language, while absolutely general truths are interpreted by means of an absolutist semantics for quantifiers (A. Rayo and G. Uzquiano, "Toward a Theory of Second-Order Consequence", *Notre Dame Journal of Formal Logic*, 40, 1999, 315–325), adapted to include a self-applicable truth predicate.

WITTGENSTEIN'S PHILOSOPHISCHER NACHLASS IST WELTKULTURERBE DER UNESCO SEIT OKTOBER 2017 – EINE KRITISCHE INSPEKTION

Josef G. F. Rothhaupt

München, Deutschland

Im Rahmen der Sitzung des Internationalen Komitees in Paris wurde per 31.10.2017 offiziell der philosophische Nachlass von Ludwig Wittgenstein in das Internationale UNESCO-Weltdokumentenregister aufgenommen. Diese Aufnahme in die Dokumentenabteilung des Weltkulturerbes ist höchst beachtenswert und äußerst lobenswert. Dass gerade der schriftliche Nachlass eines – und gerade dieses – bedeutenden Philosophen des 20. Jahrhunderts den Rang von Weltkulturerbe erreicht hat, sollte Anlass sein, sich intensiver damit zu beschäftigen. Bei der Rezeption des Wittgenstein'schen philosophischen Nachlasses sind allerdings mehrere Punkte – eben auch vorhandene Shortcomings – besonders zu beachten. In vier Schwerpunkten wird hier nun eine kritische Inspektion vorgenommen.

THE APPLICABILITY OF ARITHMETICAL CONCEPTS

Markus Säbel

Berlin, Germany

One of the great attractions of Frege's and Russell's accounts of the foundations of arithmetic was their analysis of the logical form of ascriptions of number, i.e. of the role of number words in statements like "There are three apples". While there is no explicit general analysis of such statements in the *Tractatus*, the outlines of an analysis can be extracted from Wittgenstein's remarks on the elimination of identity, his operational definition of number and his notion of a formal series. The basic idea is that ascriptions of number constitute a formal series that can be represented as being generated by the repeated application of an operation to a propositional base and that numbers serve as "exponents" of this operation. By providing an operational calculus for these exponents, the

account explains not only (in Dummett's terms) the "adjectival" use of number words in ascriptions, but also the "substantival" use in arithmetical equations and so provides the basis for a powerful explanation of the applicability of arithmetical concepts.

THE WORLD OF NECESSARY EXTENSIONS: WITTGENSTEIN'S EARLY PHILOSOPHY OF MATHEMATICS.

Giovanni Sanavio

Padua, Italy

Wittgenstein's early reflections about logic and mathematics introduced for the first time an understanding of languages as the kind of things one may live in, and of concepts as what is made use of. The sharp distinction between those entities belonging to the physical world and the ones held in the unsaid, has been probably developed with the benefit of a marked disposition of phenomenological flavor. With respect to that tradition, this brief presentation starts from the need of finding a place to Wittgenstein's concerns about mathematical structures and propositions within this broader tradition. However, what the core of this work aims to conclude is that a responsible endorsement of an intensional perspective, moving from the old categories of meaning and sign, towards intension and extension, might be the right key for making sense of many otherwise counterintuitive positions characterizing Wittgenstein's Tractarian and post-Tractarian thoughts. Across the notions of contingency and necessity, distinguishing the physical from the mathematical, numbers will be defined as pure extensions, beyond contradictions.

DEPENDENCE AND INDEPENDENCE IN LOGIC

Gabriel Sandu

Helsinki, Finland

I will argue that the most significant role of the logic of first-order quantifiers lies in its power to express functional dependencies and independencies between variables. The dependence of a variable x on another variable y has been standardly expressed by the formal dependence of a quantifier Qx on another quantifier Qy , which, in turn, is expressed by the former being in the syntactical scope of the latter. First-order logic, where scopes are required to be nested, cannot express all the possible patterns of dependence and independence between variables. To overcome this problem, two solutions have been proposed: to allow for more patterns of dependence and independence between quantifiers (Independence-Friendly (IF) logic); to express explicitly dependencies and independencies of variables (Dependence logic, Independence logic, etc). In both approaches the truth of a sentence amounts to the existence of appropriate "witness individuals" (Skolem functions). We have here a connection between the truth-conditions of quantified sentences and the existence of all the functions which produce these witness individuals. Hintikka has repeatedly argued that these functions codify winning strategies in certain (semantical) games and emphasized their connection to Wittgenstein's language games. In my presentation I will look at the interesting perspective that language games open for the discussion of logic in general. Some of these points have been discussed in J. Hintikka and G. Sandu, "What Is Logic", in D. Jacquette (ed.), *Philosophy of Logic*, Elsevier 2007, 13-39.

TWO WAYS TO THINK ABOUT (IMPLICIT) STRUCTURE

Georg Schiemer

Vienna, Austria

According to a dominant view in modern philosophy of mathematics, mathematics can be understood as the study of abstract structures. In this talk, I will compare two ways to think about the structural content of theories of pure mathematics. According to the first approach, the implicit structure or the structural properties of mathematical objects (such as number systems, groups, vector spaces, and graphs) are specified with reference to formal languages, usually based on some notion of definability. According to the second approach, structures are determined in terms of invariance criteria. For instance, the structural properties of a given mathematical system or its objects are often said to be those properties invariant under certain transformations of the system or under mappings between similar systems. In the talk, I will further investigate these two approaches to think about implicit structure in terms of invariance and definability conditions by drawing to several examples from finite geometry. Based on this, I will give a philosophical analysis of the conceptual differences between these methods and discuss their relevance for our present understanding of mathematical structuralism.

DAS „ERLÖSENDE WORT“– BEMERKUNGEN ZU ZIEL UND STIL VON WITTGENSTEINS PHILOSOPHIEREN

Alfred Schmidt

Wien, Österreich

Wittgenstein spricht nach 1929 an verschiedenen Stellen seines Werkes davon, es gehe ihm um eine neue Denkweise, eine andere Denkbewegung in der Philosophie. Er charakterisiert dieses andere Denken negativ als nicht wissenschaftlich, nicht fortschrittsorientiert und unzeitgemäß. Um das Ziel seines Philosophierens positiv zu beschreiben, verwendet er wiederholt die Wendung vom „erlösenden Wort“, das der Philosoph zu finden trachtet.

Der Vortrag ist ein Versuch der Annäherung an Wittgensteins andere Denkbewegung durch eine Interpretation, die den Ausdruck vom erlösenden Wort mit seiner bekannten Bemerkung verbindet, nur eine dichterische Darstellungsform sei der Philosophie angemessen.

FORCE, CONTENT AND LOGIC

Michael Schmitz

Vienna, Austria

The Frege point to the effect that e.g. the clauses of conditionals are not asserted and therefore cannot be assertions is often taken to establish a dichotomy between the content of a speech act, which is propositional and belongs to logic and semantics, and its force, which belongs to pragmatics. Recently this dichotomy has been questioned by philosophers such as Peter Hanks and Francois Recanati, who propose act-theoretic accounts of propositions, argue that we can't account for propositional unity independently of the forceful acts of speakers, and respond to the Frege point by appealing to a notion of force cancellation. I argue that the notion of force cancellation is faced with a dilemma and offer an alternative response to the Frege point, which extends the act-theoretic account to logical acts such as conditionalizing or disjoining. Such higher-level acts allow

us to present forceful acts while suspending commitment to them. In connecting them, a subject rather commits to an affirmation function of such acts. In contrast, the Frege point confuses a lack of commitment to what is put forward with a lack of commitment or force *in* what is put forward.

WITTGENSTEIN ÜBER DIE PHILOSOPHIE DER MATHEMATIK, 1937–1939: DIE PROJEKTIERTE FRÜHFASSUNG DER *PU* / WITTGENSTEIN'S PHILOSOPHY OF MATHEMATICS, 1937–1939: THE EARLY VERSION OF *PI*

Joachim Schulte

Zurich, Switzerland

This workshop is meant to offer an opportunity to review the development of Wittgenstein's philosophy of mathematics during the years 1937–39. The discussion will focus on TS 221 (published in my *Kritisch-genetische Edition of Philosophische Untersuchungen* as part of the *Frühfassung*) and its revised version TS 222 (published as Part I of *Bemerkungen über die Grundlagen der Mathematik = Remarks on the Foundations of Mathematics*). The basic idea is that even a rough comparison between these two typescripts may reveal some crucial features of the development of Wittgenstein's ideas during these years. In view of the fact that there is an English translation of TS 222 it should be possible to conduct great parts of the workshop in English. A survey of the extant manuscripts and typescripts will be given at the beginning (1st session). This should be a sufficient basis for raising and dealing with some questions concerning the development of Wittgenstein's ideas during this period (2nd session). The remaining sessions should be dedicated to identifying and discussing a few central notions of Wittgenstein's philosophy of mathematics during the relevant period, e.g. rule-following, logical inference, proof, calculation vs. experiment, the inexorability of logical arguments, etc. Again, the discussion of these topics will, if necessary or desirable, be introduced by brief presentations.

UNIVERSAL TRANSLATABILITY: A NONCIRCULAR JUSTIFICATION OF (CLASSICAL) LOGIC

Gerhard Schurz

Düsseldorf, Germany

The *circle of logic* refers to the fact that in order to prove the semantic validity of logical rules, one need to assume these (or other) logical rules in one's metalogic. It is well known that there are *non-classical* alternatives to classical logics. How can one justify classical logic, or a system of logic at all, in view of this situation? Is the threatening situation of an epistemic circle or infinite regress unavoidable?

The situation seems hopeless. Yet in this talk I will suggest a positive solution to the problem based on the fact that logical systems are *translatable* into each other. In my talk I present a translation method by means of introducing additional concepts into the language of classical logic. Based on this method all well-known – and I conjecture all – non-classical logics can be translated into classical logic. What this argument would show, if it is correct, is that classical logic is *optimal* in the following sense: by using it we cannot lose, because if another logic turns out to have advantages for certain purposes, we can translate and thus embed it into classical logic. Note that this optimality argument does not exclude that there can be other, non-

classical logics that are likewise optimal in the explained sense.

THE ANALOGY BETWEEN GOODMAN'S RIDDLE OF INDUCTION AND WITTGENSTEIN'S PARADOX OF RULE-FOLLOWING

Radek Schuster

Pilsen, Czech Republic

The aim of the contribution is to examine the analogy between the grue paradox, which Goodman introduces to transform the old problem of induction into the problem of projection, and Wittgenstein's paradox of rule-following, according to which every course of action can be interpreted in accordance with a rule. I argue that the analogy reveals the following findings: On one hand, if we accept Kripke's skeptical solution of the rule-following paradox we can apply it to the grue-paradox too. Moreover, it accords with Goodman's own solution strategy. On the other hand, if we deny the skeptical reading, following Baker and Hacker, and we apply the dissolution by means of conceptual clarification also to the grue paradox, we can see that the solution of both paradoxes requires more than anchoring our rules in a public domain of agreement. Since even in the public logical space we confuse "understanding" which has the internal relation to temporal "acting" with "interpreting" which has infinite potentiality. Finally, I conclude with a general proposition, drawn upon the analogy, that language enables us to express infinity in a public logical space, or more precisely, to make rules that direct us at infinity explicit. However, in this course we often tend, unwittingly, to extricate ourselves from our finiteness by confusing a temporal sense with an atemporal one in our expressions and thus we construct a paradox.

CAN EUCLID STILL BE CLASSIFIED AS AN APPLIED MATHEMATICIAN?

Dana S. Scott

Pittsburgh, Pennsylvania, USA

In 1959 Eugene P. Wigner delivered a paper under the title: "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" (*Communications in Pure and Applied Mathematics*, Vol. 13, No. 1, 1960, 1–14). It has been often reprinted and discussed numerous times every year since then. Of course, such greats such as Wigner, Einstein, and von Neumann employed very advanced mathematics for their applications, as required in discussions of dynamics and force fields and the like. By comparison, static Euclidean geometry seems very simple minded, yet many commentators today still dispute that such an "idealized theory" can be said to apply to the real world. Let us open that discussion again and see how far we can get.

PLURALITIES, GROUPS, AND PARADOX

Stewart Shapiro (joint work with Eric Snyder, Ohio, USA)

Columbus, Ohio, USA

Plural logic was originally devised by George Boolos (1984,1985) to formulate a rigorous and logically perspicuous analysis of ordinary plural talk. It purports to accomplish these goals by distinguishing two kinds of reference relations, singular reference, which is to just one object, and plural reference, which is to many objects. In

contrast, almost all semanticists recognize only singular reference, and so plurals are taken to refer to set-like things, sometimes called groups. It is common in philosophy to reject this approach, since it leads to Russell's paradox. We argue (i) that plural reference is not empirically adequate, and (ii) the threat of paradox can be eliminated if one understands groups modally, as existing only potentially – following the general account of Øystein Linnebo. Our view has some ramifications for the philosophy of set theory (and mathematics generally) and for the issue of absolute generality.

INVARIANCE AS A BASIS FOR NECESSITY AND LAWS

Gila Sher

San Diego, California, USA

Properties and relations in general have a certain degree of invariance, and some types of properties/relations have a stronger degree of invariance than others. Generalizing, we can talk about the degree of invariance of whole disciplines and clusters of disciplines (logic, mathematics, physics, biology, etc.). In this talk I will show how the degrees of invariance of different types of properties are associated with, and explain, the modal force of the laws governing them. I will then use the generalized notion of invariance to make first steps toward characterizing and explaining differences in modal force of laws/principles of different disciplines, starting with logic and mathematics and proceeding to physics and biology.

UNLOCKING HERMENEUTIC CIRCLE: FREGE VERSUS WITTGENSTEIN

Helen N. Shulga

Moscow, Russia

The paper is devoted to exploring the philosophical and historical background of discussions on the nature of contradiction among some members of the Vienna Circle in the beginning of the 1930s. The focus is not only historical but also conceptual. I examine why and how (middle) Wittgenstein could still be held as a forerunner of paraconsistency and why we should still pay attention to his prophetic remarks regarding the dawn of those systems. The normative notions of prohibition and authorization ground Wittgenstein's anti-realist view that contradictions should be thought of in terms of conflicting rules in our practices and not corresponding to any (peculiar) state of affairs in reality. This is the reason that I refer to this approach to understand paraconsistency as an anthropological approach.

MATHEMATICIANS VS. WITTGENSTEIN

Karl Sigmund

Vienna, Austria

Wittgenstein located his "chief contribution" in the philosophy of mathematics, but there are few who would agree with him. "Unorthodox" seems the most benign verdict. In this talk I will focus less on mathematics than on mathematicians: Wittgenstein's contacts (or pointed lack of contacts) with mathematicians in Cambridge or in Vienna, Wittgenstein's views on what mathematicians were actually doing (or thought they were doing), and Wittgenstein's impact (if any) on today's mathematical community.

ON GAMES, CONTRADICTIONS, AND NORMATIVITY: AN ANTHROPOLOGICAL APPROACH

Marcos Silva

Maceió, Alagoas, Brazil

The following paper is devoted to exploring the philosophical and historical background of discussions on the nature of contradiction among some members of the Vienna Circle in the beginning of the 1930s. The focus is not only historical but also conceptual. I examine why and how (middle) Wittgenstein could still be held as a forerunner of paraconsistency and why we should still pay attention to his prophetic remarks regarding the dawn of those systems. To do so, I address certain deontic notions presented in Wittgenstein's remarks and connect them to his criticism of Frege's *Grundgesetze* (1903). The normative notions of prohibition and authorization ground Wittgenstein's anti-realist view that contradictions should be thought of in terms of conflicting rules in *our* practices and not corresponding to any (peculiar) state of affairs in reality. This is the reason that I refer to this approach to understand paraconsistency as an anthropological approach.

RUSH RHEES ON WITTGENSTEIN'S PHILOSOPHY OF MATHEMATICS

Kim Solin

Helsinki, Finland

Rush Rhees is arguably the one of Wittgenstein's three trustees who has received the least attention in the secondary literature. In letters to von Wright, kept in Helsinki, Rhees describes his understanding of Wittgenstein's writings on the philosophy of mathematics. I discuss some remarks from two of these letters, and, in passing, contrast the remarks to a few other approaches to the philosophy of mathematics.

TOWARDS AN APPLICATIVE AND NOTATIONAL CONCEPTION OF WRITING PHILOSOPHY BETWEEN MATHEMATICS AND MUSIC

Antonia Soulez

Paris, France

The use of "graphematics" of a Peircian source has been noticed by Max Black in his Companion to Wittgenstein's *Tractatus Logico-Philosophicus* (see Cornell UP 1964: 2.13). The reader's sensitivity to the perceptual dimension of grasping notations has in turn gained profit from a better insight into Wittgenstein's last aspectual orientation. New ways of seeing symbolism appear along different phases of his conception of notation, showing how 1 – the tree-like schema of generating modal symbolism, rules out a Platonic genealogy stemming from a master-FRP of FRP and 2 – such schemata of family resemblances produce new embranchments in language-games in contrast with models of deductive logic. Consequently, not only resonances in language but graphics in applied mathematics and arts suggest a diagrammatic method in descriptive procedures. Hence a radical shift from the meaning of "application" connected with an *Abbildungsmethode* to a technique of "application" of symbols to a "fluent reality" requiring the use of inexact tools, that leads in turn to a reversal of the relation between models and reality so as to rule out normative models in favor of conventional constructed models binding themselves to the fluent real to which they have to adjust. The result is

sought not so much to reject scientificity, as, by dissolving analyticity, throw light on how "application" could generate ways of grasping features of reality with the help of visual charts. Assuming that Wittgenstein expects from a diagrammatic method to contribute to an applicative and notational kind of philosophy, I take a special emphasis put on an engineer-like conception of *Anwendung*, to contribute to shaping our grammatical tools.

THE STRUCTURAL COMPLEXITY OF NORMATIVE JUDGMENTS

Isidora Stojanovic

Paris, France

Evaluative judgments, such as "It is good to recycle plastics", may be distinguished from *deontic* judgments, such as "One ought to recycle plastics" (see Christine Tappolet, "Evaluative vs. Deontic Concepts", in H. LaFollette (ed.) *International Encyclopedia of Ethics*, Wiley-Blackwell, 1791–99).

Both evaluative and deontic judgments are considered to be *normative* judgments, as opposed to *descriptive* judgments, such as "Most people recycle plastics". The distinction between deontic and evaluative judgments relates to the distinction between norms and values (see Sven Ove Hansson, *The Structure of Values and Norms*, Cambridge University Press, 2007).

However, neither distinction is without controversy, and both remain largely underexplored. According to a recent proposal from Olivier Massin, (see "Desire, Values and Norms", in J. Deonna and F. Lauria, *The Nature of Desire*, Oxford University Press, 2017), the two distinctions rely ultimately on certain logical differences between the two types of judgment.

In this talk I critically examine Massin's proposal by assessing the logical differences he identifies from the standpoint of scalar semantics for gradable predicates.

UNDERSTANDING AND EXPERTISE

Barry Stroud

Berkeley, California, USA

Wittgenstein says that "when one has attained greater clarity about the concepts of understanding, meaning something, and thinking ... it will then ... become clear what may mislead us (and did mislead me) into thinking that if anyone utters a sentence and means or understands it, he is thereby operating a calculus according to definite rules" (*Philosophical Investigations* §81). This paper takes up the questions (1) why someone's correctly applying a concept he understands to particular cases is not to be accounted for by his "operating a calculus according to definite rules", and (2) how his correctly applying the concept to particular cases is to be explained by the competence involved in his understanding the concept. It is because he understands it that he applies the concept correctly. These questions are relevant to the "paradox" of *Philosophical Investigations* §201.

WHAT HILBERT AND BERNAYS MEANT BY "FINITISM"

William W. Tait

Chicago, Illinois, USA

"Finitism" (W. Tait, *Journal of Philosophy*, 78, 1981, 524–556) presents an argument that finitist number theory is primitive recursive arithmetic (PRA). The argument is based on taking seriously the "finite" in "finitism". But the

question remained: what did Hilbert (and Bernays) mean in the early 1920's through the early 1930's by "finitism" and in particular, did they restrict finitist number theory to PRA. In his dissertation ("Numbers and Functions in Hilbert's Finitism", *Taiwanese Journal of Philosophy and History of Science*, 10, 1998, 33–60), Richard Zach pointed out that Hilbert endorsed results as finitist that require more than PRA for their proofs. "Remarks on Finitism" (*Reflections on the Foundations of Mathematics: Essays in Honour of Solomon Feferman*, A K Peters/CRC Press, 410–19) and *The Provenance of Pure Reason: Essays in the Philosophy of Mathematics and Its History*, Appendix (Oxford University Press, 2005) argue that it is not clear that Hilbert was aware that these results go beyond PRA. But that view is challenged in more recent times in W. Sieg and M. Ravaglia, "David Hilbert and Paul Bernays, *Grundlagen der Mathematik*" (I. Grattan-Guinness (ed.) *Landmark Writings in Western Mathematics, 1640–1940*, Elsevier, 2005, 981–999) and by the editors of (the invaluable!) *David Hilbert's Lectures on the Foundations of Arithmetic and Logic 1917–1933* (M. Hallett et al. (eds.), Springer, 2013). I will survey the old ground and then discuss the new challenge, which claims that, from the early 1920's on, Hilbert accepted as finitist an enumeration function of the primitive recursive functions (which of course is not primitive recursive). The grounds for this are a reading of a passage in §7 of *Grundlagen der Mathematik I* and an argument for the consistency of PRA which goes back to 1922–3 and is elaborated again in §7 of *Grundlagen der Mathematik I*. I will argue that their reading of the passage in question is a misreading and that the argument for the consistency of PRA uses, not an enumeration function for the primitive recursive functions, but rather mathematical induction on a Π_2 predicate (i.e. of the form $\forall x \exists y \varphi(x, y)$), which was explicitly rejected by Hilbert as finitist – e.g. notably in "Über das Unendliche" (*Mathematische Annalen*, 95, 1926, 161–90).

"A SPATIAL OBJECT MUST LIE IN INFINITE SPACE" (*TRACTATUS*, 2.0131)

Shunichi Takagi,
London, UK

At an early stage of the *Tractatus*, Wittgenstein claims that 'a spatial object must lie in infinite space'. In addition to an initial task of clarifying what this thesis says, it also raises a question of what it does in the context of giving the general form of proposition. As an answer to this question, I argue a main purpose of this thesis lies in presenting an unbounded conception of space such that what is thinkable about spatial objects are not restricted by some contingent, empirical elements. By endorsing this thesis, I shall show, Wittgenstein was opposed to Russell's conception of space expressed in *Our Knowledge of the External World* and other works since Russell's view mistreated the nature of possibility. On the basis of these, I shall suggest that this issue is connected, both philosophically and historically, to a broader issue of Wittgenstein's philosophical commitment *vis-à-vis* Kant's transcendental Idealism.

CONTRADICTIONS AND MATHEMATICAL ERRORS

Harry Frederick Christian Tappenden
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Wittgenstein said at several points that contradictions in inconsistent logical or mathematical systems cannot cause physical failures, such as the collapsing of bridges. I find there to be a tension between this and his claim that

mathematical errors can cause such things, and I conclude that Stuart Shanker's emphasis on Wittgenstein's view that contradictions cannot be incorrect, and that they can. The crux of the matter is that contradictions, whilst often obstructions in a calculation, do not have to be, and for any calculation that continues from them, one can of it that the subsequent physical failure was, in part, due to the contradiction involved, just as one could say so of a mathematical error elsewhere.

WITTGENSTEIN ÜBER DAS PROBLEM DER VERSTELLUNG UND DEN EUKLIDISCHEN RAUM: „ICH MEINE: ES SEI EINE TIEFGEHENDE ÄHNLICHKEIT VORHANDEN“

Jasmin Trächtler
Bergen, Norwegen

In seinen *Bemerkungen über die Philosophie der Psychologie* und *Letzten Schriften zur Philosophie der Psychologie* (1946–1951) widmet sich Wittgenstein verschiedenen Belangen der menschlichen Seele, wobei seine Gedanken vielfach auch um das Fremdseelische kreisen, mit seinen Tücken der Lüge, Verstellung und Heuchelei. Denn es sind insbesondere diese Tücken, aufgrund derer die fremde Seele zum philosophischen Problem wird – ein Problem, das Ausdruck findet in dem skeptizistischen Einwand, dass „man nie wissen könne, was im Anderen vorgeht, da alles immer auch Verstellung sein könne“. Diesem skeptizistischen Einwand sucht Wittgenstein mit einer grammatischen Charakterisierung desselben beizukommen, im Zuge derer er zweimal eine „tiefergehende Ähnlichkeit“ zwischen dem Problem der Verstellung und dem Verhältnis des Euklidischen Raums zur Gesichtserfahrung andeutet. Ziel des Vortrags ist es, dieser etwas rätselhaft anmutenden Andeutung nachzuspüren und anhand dessen zu zeigen, dass der skeptizistische Einwand auf einer sprachlichen Missdeutung des „man kann nie wissen...“ beruht und das Problem der Verstellung daher weniger erkenntnistheoretischer, als vielmehr praktisch-moralischer Natur ist.

PHENOMENOLOGICAL APPROACH TO INFINITY AND CONTINUUM

Kateřina Trlifajová
Prague, Czech Republic

Since the 1960s, when Robinson non-standard analysis was established, several other non-standard models of natural and real numbers have been created. The not widely known theory of the Czech mathematician Petr Vopěnka, Alternative Set Theory, AST, was also developed.

It is an alternative to Cantor's Set Theory, which Vopěnka criticized for numerous reasons. Cantor's justification for accepting the actual infinity was theological; in modern axiomatic systems it is expressed by the axiom of infinity. Infinite hierarchy of infinite cardinal and ordinal numbers finds minimal interpretation in the real world. The existence of independent theorems leads to dividing set theory into several branches, from which none can be considered the sole truth. Vopěnka's AST relies on phenomenology and endeavours to interpret basic terms of infinite mathematics in the real world. It uses the infinite for the mathematization of indistinctness. Apart from classic sets and classes, here so-called semisets are introduced.

AST can be partially formalized as the non-standard model. Similarly, as with other non-standard theories, it does not bring breakthrough mathematical results that have been impossible to describe in a standard manner.

What is substantial is its philosophical interpretation, which attempts to retain correspondence with the real world. It offers the solution of certain old philosophical problems: Zeno's paradoxes, *sorites*, Leibniz's conception of continuum, Pascal's double infinity.

EXPERIMENTS IN MATHEMATICS AND THE NATURAL SCIENCES – THE ANALOGY WITHIN THE DESCRIPTIVE EPISTEMIC CONTEXT

Majda Trobok

Rijeka, Croatia

In this paper my aim is to show that, given the descriptive epistemic context, the analogy between mathematics and the natural sciences holds even when one's epistemic route is experimentation. Experiments are usually taken to be the lynchpin of the natural sciences investigations, which seems to be the domain that does not have much in common with mathematics and the way we grasp the basic mathematical concepts. Putnam, on the other hand, interestingly points out that experiments are perceived in the mathematical investigations too. My goal is to go one step further than Putnam might have wanted to go and, hopefully, show that the analogy holds throughout.

ZYKLIZITÄT DER IMAGINÄREN ZAHLEN

Walter Tydecks

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Die imaginären Zahlen können neben der Differentialrechnung als der größte eigenständige Beitrag der westeuropäischen Mathematik angesehen werden, vergleichbar der Einführung der Zahlen bei den Ägyptern und Sumerern und der Euklidischen Geometrie in der Antike. Sie gelten jedoch meist als rein technische Leistung, wie aus negativen Zahlen Wurzeln gezogen werden können, und ihr Erfolg in der Quantenphysik erscheint selbst Mathematikern und Physikern kaum verständlich und geheimnisvoll. Abgesehen von wenigen Stellen bei Cassirer und Quine blieben sie in der Philosophie unbeachtet, und nur ein Außenseiter wie der Wittgenstein-Schüler Spencer-Brown hat erkannt: Die Zyklizität der imaginären Zahlen kann in der Philosophie und Logik eine vergleichbare Wende auslösen, so wie aus der Transitivität der gewöhnlichen Zahlen die Kausalität und das kausale Denken hervorgingen, aus der Axiomatik Euklids die Architektur aufklärerischer Systeme und aus den Grenzübergängen der reellen Zahlen das Prinzipien-Denken seit Leibniz und der Begriff der höheren Unendlichkeit (Kontinuität) bei Hegel.

Der Vortrag gliedert sich in drei Teile:

Im ersten Teil gebe ich einen Überblick über die Geschichte der imaginären Zahlen. Der zweite Teil präsentiert die überraschenden Lösungsvorschläge von Spencer-Brown zur Russellschen Antinomie und dem Lügner-Paradoxon. Der dritte Teil skizziert den Entwurf eines Zyklizitäts-Denkens.

TARSKI'S ACCOUNT OF LOGICAL CONSEQUENCE: A DEFENCE

Shota Uka

Salzburg, Austria

After giving a precise definition of truth in his paper "The concept of truth in formalized languages", published in 1933, just three years later Tarski set out to do the same for the concept of logical consequence, employing the

concept of satisfaction by a sequence which he successfully used in defining the notion of truth. According to this definition, a sentence φ is a logical consequence of a set of sentences Γ iff every interpretation that makes all sentences of Γ true also makes φ true.

Although Tarski's definition of logical consequence is widely accepted, Etchemendy argued in his book "The Concept of Logical Consequence" that Tarski's account is both conceptually and extensionally inadequate. In his view Tarski's account fails to capture the essential features of logical consequence. As a result it gets the wrong results, because either it overgenerates or it undergenerates. And if it gets the right results, this is not because it is "testing for the right thing".

In my talk I will quickly introduce Tarski's definition of logical consequence, followed by a brief presentation of Etchemendy's criticism. My aim is to explain what goes wrong in Etchemendy's critical analysis of Tarski account, trying to show (1) why Etchemendy's epistemological argument rests on false assumptions, and (2) why the counterexamples Etchemendy gives against Tarski's account are no counterexamples against it. I will argue that Etchemendy's criticism is built on a misunderstanding of Tarski's aims and a mischaracterization of his account.

WITTGENSTEIN AND THE PROBLEM OF WILL IN PHILOSOPHICAL INVESTIGATIONS

Miroslav Vacura

Prague, Czech Republic

This paper focuses on Ludwig Wittgenstein's reflections on the nature of the will as presented in *Philosophical Investigations*. When Wittgenstein first encountered the problem of will, opinions on the issue seemed to be polarized between the ideas of empiricists and those of Schopenhauer. Firstly, empiricist account of the will will be examined; thereupon a brief investigation into current developments of empiricist ideas, particularly within cognitive science, will be undertaken as the basis of further discussion. In subsequent paragraphs, it will be demonstrated that the most fruitful results cannot be obtained from observations of the usual manifestations of will, but from an analysis of failed actions; several prominent examples have been provided by Wittgenstein. Lastly, it will be argued that while Wittgenstein opposed empiricist lines of thought, his philosophy may be reconciled with them in the light of results from contemporary philosophy.

INTUITIONISTIC INFINITE PROOFS

Mark van Atten

Paris, France

Brouwer viewed proofs as mental objects, ontologically, and, as infinite objects, in general ("Über Definitionsbereiche von Funktionen", *Mathematische Annalen*, 97, 1927, 64 n.8).

He defined the relation between finite linguistic proofs and infinite mental proofs by appealing to a notion of canonical form: linguistic proofs are non-canonical representations of mental proofs, and can be turned into them by a canonisation procedure. Brouwer exploited this in a controversial proof of the Bar Theorem.

In this talk, which aims to clarify a number of distinctions, I pose some Wittgensteinian questions about this view:

(1) Would the infinity of a mental proof be the infinity of a rule, or the infinity of an extension? (2) How could proofs be infinite objects given that they would then be unsurveyable, and their conclusions unreachable? (3) Would we know that a non-canonical proof is indeed a

proof because the corresponding canonical proof is itself one – is the former but a “pale shadow” of the latter?

(4) If proof is an open-ended notion, how could we justify the claim that every future proof of a given proposition will yield to a canonisation procedure we are now in position to propose?

ASSERTION AND GROUNDING

Maria van der Schaar

Leiden, The Netherlands

Is assertion essential to logic? Assertion plays a central role in logic if one understands logic as the theory of assertion and (epistemic) inference (Per Martin-Löf, “Is Logic Part of Normative Ethics?”, Lecture Utrecht, 16 April 2015 & Paris, 15 May 2015). In addition, for a realist like Frege, the judgement stroke, indicating assertive force, is essential to logic. But does the presence of the judgement stroke make one’s logic psychologistic?

If one makes an assertion, an interlocutor is entitled to ask “How do you know that?”. One thus needs to be able give a ground for one’s assertion. Will any grounding do? Some grounds merely give the *ratio cognoscendi* (the epistemic ground); others also give the *ratio essendi* (the reason why). The assertion of the former answers the question “How do you know that?”, whereas the expression of the latter answers the question “Why is that true?” Both the idea of logic as a science and the idea of logic as a theory of inference demand an answer to the latter question.

If we ask the question why something is true, grounding needs to end somewhere. Does it end in self-evident first principles? Or is self-evidence merely a psychological notion, and does our grounding end, instead, with the hinges on which our questions and answers turn?

LIVING IN THE WORLD OF POST-NON-CLASSICAL LOGIC

Vladimir L. Vasyukov

Moscow, Russia

A new Tower of Babel was the project of developing a unique and uniform logic supposed to provide rules of correct reasoning for all. Although this attempt has been deemed successful for over two thousand years it has ultimately failed as a result of the development and proliferation of so-called “non-classical logics”. As a result, many of today’s logicians are logical pluralists, holding the view that there are many alternative systems of logic rather than one “right” logic. It may appear that logical monism needs no justification because it is supported by two thousand years of evidence in the history of logic.

The situation is not, however, so simple, not least because the arguments offered by logical monists are often ultimately ethical or aesthetic justifications rather than properly logical ones. Monists call for a return to the “lost paradise” from which earlier logic and logicians have been expelled. The existing experience of metalogical research indicates, however, that there is no single logical system capable of delivering all the required metalogical properties and which is, simultaneously, free from all paradoxes. Moreover, it is difficult to identify even a short list of universal meta-properties which such an ideal logical monist system should necessarily possess.

These facts provide additional evidence in favor of the post-non-classical view according to which a logician should select his/her formal toolkit on the basis of those particular social goals, values and norms which dictate the his/her choice of research strategy.

FORM OF LIFE: THE ORDINARY THREATENED

Kristina Veinbender

Prague, Czech Republic

The concept of *forms of life* is put forward by Wittgenstein out of the conviction that philosophy cannot be based on what goes beyond the *ordinary*. The goal of this paper is to pursue the question whether this represents a frame of philosophical inquiries such that philosophy is capable to give answers that can last or are immune to change or even tell us what is necessarily so. By following the writings by Evans-Pritchard, Stanley Cavell and Peter Winch, I will show that to answer this question requires to have a clear (over)view of what is “outside” or “inside” a *form of life*. As I will suggest, even skeptical threats to what seems for us irreversible conditions of our practice are not to be seen as a questioning that comes “from outside”. Such a skeptical practice is rather intrinsic to human beings *forms of life*. But should the consequence then be that philosophical views also rest on instable conditions? Or, would to draw such a consequence in fact be a equivocation of what is “inside” and “outside”?

TURNING POINTS IN THE FOUNDATIONS OF ARITHMETIC, FROM GRASSMANN TO GÖDEL

Jan von Plato

Helsinki, Finland

Modern foundational research in mathematics has two-fold roots, geometric and arithmetic. In the latter, the decisive point was Hermann Grassmann’s discovery of the recursive definitions of the basic arithmetic operations in 1861. It opened the way to the inductive proofs of the laws of arithmetic, earlier thought to be axioms. Peano followed Grassmann in his axiomatisation of arithmetic, but at this point, around 1890, the idea of a “doctrine of numerical quantity” (*Anzahllehre*) was also prevalent. Frege was its most extreme proponent, others such as Dedekind and Husserl were somewhat eclectic, trying to get the best out of both of these main approaches to the foundations of arithmetic. Peano’s work had a grave defect, namely, his logic contained next to propositional logic only half of the principles of reasoning with the quantifiers. In particular, he failed to understand Frege’s rule of generalisation. In Russell’s *Principia*, this was corrected. In the end, Grassmann’s inductive way became the prevalent one, through Skolem’s development of primitive recursive arithmetic published in 1923. It was followed by Ackermann and Bernays with full formalizations of first-order arithmetic in the latter part of the 1920s. The hopes were high that these systems could be proved complete. Following an idea of Skolem’s, Gödel saw that no such formalization is possible. In the presentation, Gödel’s work will be portrayed in part through its background in the research into foundations of arithmetic, in part through the notes Gödel wrote on his way to the final formulation of the incompleteness theorem. These notes are found among the papers Gödel left behind, included in a microfilm edition. They were written in shorthand in the summer and early autumn of 1930 and are here examined for the first time.

WITTGENSTEIN ON NONSENSE

Chong Wang

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In this paper, I will be concerned with different attempts to account for Wittgenstein’s understanding of *nonsense*. There are the ones who believe that sentences become

nonsensical, if the conditions of use are not honored. So called "philosophical nonsense" comes from ignoring the role of the context of an utterance plays. But what exactly is meant by "ignoring the role of context"? What is it about "the context of use" that could be said to be the reason why certain questions or answers – that look to be meaningful – do in fact lack any meaning? Does failing to honor the context just mean a particular sentence would never be uttered in ordinary life? Or does it mean it would never be uttered because all sentences in order to be meaningful must *share* something – something alike a "logical form"? Being an example of nonsense would in this respect represent a violation of conditions that could be said to be "conditions of logic". This is the other view that has been expressed in relation to Wittgenstein on *nonsense*.

Unlike most other philosophers, I want to argue that there are *no* features that could be pinned down as features *all* contexts or *all* utterances *must* share. This has the consequence that we cannot look at language as something completed. A "language-game" needs to be kept playing. In other words, a sentence is meaningless, if no one is capable to respond to it with understanding.

MOTIVATING WITTGENSTEIN'S MATHEMATICS

Lawrence S. Wang

Colchester, UK, and Freiburg i. Br., Germany

Wittgenstein's finitist and anti-foundationalist mathematics problematizes mathematical development; a general appeal to intuitionism is unsatisfactory. Investigating the concession of "mathematical stimuli" by which propositions motivate their legitimization through mathematics, I track a binary action where (1) an exterior proposition comes under mathematical consideration (2) this proposition is transfigured into a functionally mathematical form. While (2) extends a modal question – the "mathematical" use of a proposition relies on its assumption/translation into Wittgenstein's "geometric" grammar – (1) claims to that translatability outright. Purely normative mathematics thus abridges the stimulative action of the proposition, which rises based on descriptive measures; meaning here is not thematic but tonal, whereby any sense of normative ascription must derive from the same pictorialism as the hypothetical geometry mathematics. This promotes deeper inquiry into "intuitionism", which traces the affective quality preceding mathematical translatability to Wittgenstein's perceptive philosophy, evident across his linguistic, mathematical, and poetic philosophies.

ON THE NON-NORMATIVITY OF LOGIC

Heinrich Wansing

Bochum, Germany

Quite a few philosophers have claimed that logic is normative. Some have given accounts of logic's supposed normativity, some have explored ways in which logic might be normative, and some have rejected the view that logic is normative.

Any discussion of logic's normativity or non-normativity remains difficult, however, because the question as to whether logic is, or is not, normative needs to be made more precise. Among the extensive literature on this subject, F. Steinberger's entry in the *Stanford Encyclopedia of Philosophy* ("The Normative Status of Logic", 2017) provides a useful survey, charting the major theories and theses on the normative status of logic.

In this talk I discuss two recent papers on the non-normativity of logic: Y. Shramko, "The Logical Way of

Being True: Truth Values and the Ontological Foundation of Logic" (*Logic and Logical Philosophy* 23, 2014) and G. Russell, "Logic isn't Normative" (*Inquiry*, forthcoming).

Whilst Shramko brings into focus an ontological, realistic account of logic inspired by Frege, Russell concentrates on the ideas that entailment is truth-preserving and that there is an entanglement of logic with the normative. According to her, however, logic is entangled with the normative only in a very weak and non-distinctive way.

I propose an informational, non-epistemic understanding of logic, in which the entanglement of logic with the normative disappears.

EINE KRITIK DER MENGENLEHRE

Viktor Weichbold

Innsbruck, Österreich

Die Mengenlehre gilt schon seit der ersten Hälfte des 20. Jahrhunderts als *die* Grundlage der Arithmetik. „Set theory is the foundation of mathematics. All mathematical concepts are defined in terms of the primitive notions of set and membership". (Kunen *Set Theory. An Introduction to Independence Proofs* 1980: xi) Ihre eigenen Grundlagen sind allerdings bezweifelbar. Viele Grundbegriffe der Mengenlehre sind mehrdeutig oder widersinnig. In diesem Vortrag werde ich mich deshalb zuerst dem Begriff „Menge“ zuwenden. Gibt es „einen“ oder „den“ Begriff der Menge der den „starting point“ bilden könnte, von dem ... aus es möglich ist, „ultimately to derive all branches“ der Mathematik? (Burgess *Rigor and Structure* 2015: 60). Ich denke nicht. Ich meine auch, dass in der Idee von „extensionalen Definitionen“ die Quelle von Widersprüchlichen zu sehen ist, die sich auf tun wird der Begriff „Menge“ zur Bestimmung von arithmetischen Operationen verwendet wird. Dazu werde ich unterschiedliche Arten oder Auffassungen von „Definition“ vorstellen, diese vergleichen und die Problematik der „extensionalen Definition“ aufzeigen, mithin aber auch versuchen, eine Alternative zu der mit dieser „extensionalen Definition“ fehlerhaften Ontologie arithmetischer Begrifflichkeiten zu entwickeln. In meiner Überlegungen werde ich mich an den Analysen Benacerrafs (Benacerraf 1965: „What numbers could not be“) und Bealers *Quality and Concept* (1982) orientieren.

ON THE APPLICATION OF AXIOMATIC THINKING TO RELIGION

Paul Weingartner

Salzburg, Austria

The purpose of the paper is to show that axiomatic thinking can also be applied to religion provided a part of the language used in religion (here called: Religious Discourse) consists of propositions or norms. How it can be applied to mathematics Hilbert outlined in his "Axiomatisches Denken" (1918).

The first part of this article discusses the possibility of applying axiomatic thinking to religion by considering the necessary preconditions to be satisfied for a successful application. The second part discusses the specific logical language that will be used in the application. The third part offers three concrete examples of such an application: a partial and preliminary version of an axiomatic theory of omniscience, omnipotence, and moral evil.

KANT ÜBER SCHÖNHEIT UND ZWECKMÄßIGKEIT IN DER MATHEMATIK.

Christian Helmut Wenzel

Taipei, Taiwan

Kann Mathematik schön sein? Gibt es Leben in der Mathematik? In der *Kritik der Urteilskraft* (1790) untersucht Kant Prinzipien der Zweckmäßigkeit, eine subjektive Zweckmäßigkeit für die Ästhetik und eine objektive Zweckmäßigkeit für die Teleologie. Die Mathematik aber fällt bezüglich beider durch. Mathematische Gegenstände und Eigenschaften können nach Kant nicht schön sein und bei Erklärungen müssen wir keine Vorstellung von einem Zweck voraussetzen, denn wir können die Gegenstände konstruieren, meint Kant. Jedoch räumt er ein, mathematische „Demonstrationen“ könnten schön sein. Dies hängt mit seiner Unterscheidung von Anschauung und Begriff zusammen. Ich werde diesbezüglich einige seiner Ausführungen darstellen und problematisieren.

DAS UNENDLICHE IM MATHEMATISCHEN ALLTAG

Franz Winkler

Linz, Österreich

Das Unendliche hat eine lange und wechselhafte Geschichte in der Mathematik. Das Problem der griechischen Philosophie und Mathematik mit dem Unendlichen drückt sich aus in den Paradoxien von Zeno von Elea. Andererseits gab Archimedes einen unendlichen Prozess an, um den Umfang eines Kreises zu bestimmen. Wirklicher Fortschritt im Umgang mit dem Unendlichen wurde erst im 19. Jahrhundert gemacht mit der Theorie der Konvergenz und der Einführung der Mengentheorie von Georg Cantor. Dabei wurde gleich eine unendliche Hierarchie von Unendlichkeiten eingeführt. David Hilbert hatte keine Scheu, mit unendlichen Objekten umzugehen, und über sie Beweise zu führen, ohne konkret Elemente dieser unendlichen Mengen zu konstruieren. Ja er war der festen Überzeugung, dass jedes mathematische Problem sich letztlich auch lösen lässt. Die Resultate von Gödel und Turing zeigen uns aber, dass die Mathematik in ihrer Unendlichkeit sich letztlich immer unserem Zugriff entzieht. Die heutige Mathematik geht selbstverständlich mit unendlichen Mengen um. Wir geben dazu Beispiele an. Inwieweit solche unendlichen Mengen nur potentiell oder doch real existieren, wird in der Mathematik nicht beantwortet. Das bleibt eine Frage der Philosophie.

ON THE RELATION OF LOGIC TO METALOGIC

Jan Woleński

Kraków, Poland

We can define logic axiomatically or via rules (i.e. involving natural deduction, sequents).

The first approach also requires rules of inferences, unless all logical theorems are taken as axioms. (I omit this strategy henceforth.)

Given deduction-completeness theorems, both definitions of logic are equivalent. Take the simplest case: if the formula $A \Rightarrow B$ is a logical theorem, then the rule $A \vdash B$ is logically valid (i.e. truth preserving).

Usually, we think of logic as a theory formulated in the object-language which consists of theorems and inferential rules. However, the answer to the question “What is logic?” depends on metalogic. For instance, if we say that logic should be complete (every logical validity is provable), we

exclude higher-order logics. Here the Henkin view of completeness is not helpful.

A more sophisticated approach refers to Lindström characterization results. Hitherto, I considered classical logic. Moving beyond classical logic, the situation in non-classical logic becomes much more challenging. One example is the profound question as to whether or not the fundamental meta-theorems of intuitionistic logics are provable intuitionistically.

The general picture suggests that even if a given logic is formalized, its metalogic is partially intuitive and ultimately depends on the very decisions one makes.

PROGRAM EXPLANATIONS AND REVERSE MATHEMATICS

Krzysztof Wójtowicz

Warsaw, Poland

In this paper, the problem of the explanatory virtues of mathematics is discussed, in particular in the context of program explanations (where theorems are understood as kinds of modal constraints on the world). I assume, that – if mathematics indeed has some explanatory virtues – then it is also (partially) inherent in the proofs. So, understanding the proofs becomes crucial if we want to account for the explanatory role of mathematics (which is especially important in the context of the Enhanced Indispensability Argument (EIA). A better understanding of the nature and the role of the proof involves – in particular – examining the strength of the necessary “mathematical machinery”. And here, results in Reverse Mathematics might be important to the discussion, as it gives a precise measure of the strength of the assumptions necessary in order to prove particular mathematical theorems. However, the question of the explanatory character of these proof remains to be discussed.

PROOFS, APPLICATIONS, AND MEANING IN REMARKS ON THE FOUNDATIONS OF MATHEMATICS

Tianyu Wu

Irvine, California, USA

In this paper, I would like to show that Wittgenstein in *Remarks on the Foundations of Mathematics* makes the following points concerning meaning in mathematics: first, the meaning of a mathematical proposition depends on both the internal and external uses of that proposition; second, the relationship between the two forms of uses in determining the meaning of a proposition is a kind of means-end relationship, where internal uses function as the means, and the external uses function as the end; third, by appealing to the role external use plays, we could find ways to resolve the problems incurred by only taking meaning to be equivalent to internal use, at least in some cases; last, to obtain a fully coherent picture of meaning in mathematics, we need to further clarify how Wittgenstein could reconcile his non-revisionism with his suspicion towards pure, especially foundational, mathematics.

THE PHILOSOPHICAL SIGNIFICANCE OF THE CURRY-HOWARD ISOMORPHISM

Richard Zach

Calgary, Canada

The Curry-Howard isomorphism relates systems of natural deduction (logic) and systems of typed lambda calculi (computation). It observes that typed lambda terms (such as $\lambda x:A. y:B$) can be assigned to proofs of logical formulas (such as $A \rightarrow B$) in such a way that the formula proved correspond to type assignments of the corresponding terms, and normalization of such proofs correspond to steps in the evaluation of the corresponding terms. The correspondence was first observed by Curry in the 1930s and Bill Howard in the 1960s. It remained of mainly logical interest until the 1980s, when it was discovered that it can play a crucial role in giving semantics to functional programming languages and provide a foundation for type systems. Type systems, and notions such as type safety, which the Curry-Howard isomorphism provides, in turn are now recognized to be important properties of programming languages. This means that it has a certain practical significance in the theory of programming languages. I will argue that this practical significance is also of philosophical significance. I will do so by articulating a view of program safety by analogy to the notion of proof correctness. The Curry-Howard correspondence can then be seen to play a similar role to proofs of consistency and conservativity in the philosophy of mathematics.

“EINE ART RELATIVITÄTSTHEORIE DER SPRACHE” – WITTGENSTEIN AND EINSTEIN

Pascal Zambito,

Cambridge, UK

This paper explores the relation between aspects of Wittgenstein's philosophy and Einstein's methodological insights in physics. By investigating the references to the

theory of relativity in the *Nachlass* an analogy takes shape, an analogy which is “not arbitrary” (MS109: 58) and reveals a latent kinship between the two thinkers. It concerns three general aspects of Wittgenstein's philosophy: first, the role of verification in the grammar of propositions; second, his stance on intentionality, what he calls the harmony between thought and reality; and third, the status of propositions which are neither straightforwardly empirical nor arbitrary definitions, but seem to feature an absolute certainty. In all three domains the relativistic perspective, particularly its focus on methods of measurement instead of metaphysical objectivity and absoluteness, can shed light on Wittgenstein's method of grammatical investigation.

THE RELATION BETWEEN MATHEMATICS AND PSYCHOLOGY IN WITTGENSTEIN'S LATER PHILOSOPHY

Ligeng Zhang

Beijing, China

In this paper I deal with Wittgenstein's conceptions on philosophy of psychology and the foundation of mathematics. I begin with a remark in the second part of *Philosophical Investigations* which alludes to the similarity between psychology and mathematics, and go on to elucidate Wittgenstein's criticism on the misconceptions hold by some psychologists and mathematicians. I argue that, on the one hand, some psychologists and mathematicians are likely to misunderstand the foundation on which their investigations are built; on the other hand, a Wittgensteinian clarification on the fundamental concepts in psychology and mathematics can be seen as the “foundations” for the two fields.

Workshop 1

WITTGENSTEIN'S PHILOSOPHY OF MATHEMATICS, 1937–1939: THE EARLY VERSION OF *PI*

Org. by **Joachim Schulte**
Zurich, Switzerland

Workshop 2

LOGICAL PARADOXES

Org. by **Hannes Leitgeb**
Munich, Germany

With **Volker Halbach** (Oxford, UK), **Lavinia Picollo** (Munich,
Germany) & **Lorenzo Rossi** (Salzburg, Austria)

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überlebt.

Raiffeisen



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